Submitted to *Management Science*
manuscript (Please, provide the manuscript number!)

Allocating marketing effort under customer discrimination

Customers can be discriminatory when making purchasing decisions. For example, a White customer may prefer to buy products from White sellers. Firms can amplify or curtail customer discrimination through their decisions; for example, by promoting White sellers to appeal to White customers to maximize profit at the cost of underrepresenting certain groups of sellers. Many social commentators now agree that hiding behind the adage "the customer is always right" does not justify amplifying customer discrimination.

We look at the problem of a platform managing customer discrimination among many sellers and customers. The platform has a marketing budget and must decide how to allocate its marketing effort among sellers. Using a multinomial logit choice model, we show that a platform whose objective is to maximize the sum of seller profits makes optimal choices that may amplify customer discrimination. The optimal strategy is to concentrate all marketing effort on a single seller. We show that sellers that are highly discriminated against are unlikely to be the beneficiaries of this concentration.

We explore two ways a platform may adjust the structure of its marketing allocation problem (away from maximizing total profits) to curtail customer discrimination. First, the platform may face social pressure to constrain its allocation of marketing effort so that a minimum proportion of market share goes to a group of protected sellers. In this case, we show that the platform allocates marketing effort to at most two (and, in many cases, only a single) protected sellers. Thus, constraining market share leads to a practice similar to "tokenism" in hiring practices, where very few members of a protected group receive all the benefits intended to alleviate the harms borne by an entire population. Second, we consider a scenario where a platform has inequality-aversion preference, which is represented by the notion of α-fairness (as introduced by Atkinson (1970)). We show that this results in an allocation of marketing effort that includes more sellers, and as aversion to inequality increases (via larger values of α), marketing effort is eventually allocated to all sellers, starting with those who are more likely facing customer discrimination.

Key words: customer discrimination, inequality aversion, marketing strategy

1. Introduction

When we think about discrimination (by gender, race, age, sexual orientation, etc.) in business, the most common mental image is of a large company discriminating against its customers or employees. However, customers can be a source of discrimination at every step of a market transaction. This *customer discrimination* can occur even when companies attempt to remain neutral (Bartlett and Gulati 2016).

Let's examine some of the dimensions of customer discrimination. First, customers discriminate among the sellers they can potentially transact with. For example, Bar and Zussman (2017) find that a significant share of Jewish customers prefer to receive labor-intensive services from firms employing Jewish rather than Arab workers. Second, customer willingness-to-pay can reveal discriminatory views. In a field experiment on eBay, Ayres et al. (2015) find that baseball cards displayed in a hand with dark skin sold for approximately 20% ($0.90) less than cards displayed in a light-skinned hand. Third, customers discriminate in their evaluation of goods and services. For example, a wide array of empirical evidence indicates that students are biased in their evaluations of female instructors (Arbuckle and Williams 2003, Mengel et al. 2019, Fan et al. 2019).

While customer discrimination is certainly not a new phenomenon, today's economy offers even more opportunities for customers to discriminate. This is apparent with the expansion of platforms, where the supply side consists of individual providers rather than large companies. For example, on Airbnb, a short-term home-stay platform, the hosts are individual property owners; on Lyft, a ride-sharing platform, the service providers are individual car owners; and on Uber Eats, a mobile-enabled food delivery business, most suppliers are family-owned restaurants. Decentralized platforms have created millions of job opportunities, allowing individuals to easily reach a large customer base. However, this decentralization also makes individual suppliers vulnerable to discrimination by customers, as they have to interact directly with customers of various backgrounds. Personal information such as gender or race is often hard, if not impossible, for individual suppliers to conceal. Many two-sided platforms, such as Airbnb, encourage buyers and sellers to disclose personal information, including pictures, to build trust and facilitate transactions. On the positive side, this information allows customers to learn not only about the products or services offered but also about the individuals providing them. On the negative side, such information can facilitate customer discrimination based on the seller's race, gender, or other personal aspects.

Studies have identified the existence of racial disparity on two-sided platforms. Edelman and Luca (2014) show that non-Black hosts on Airbnb charge approximately 12% more than Black hosts for the equivalent rental. Cox (2017) shows that across all 72 predominantly Black New York City neighborhoods, White Airbnb hosts earned 73.7% of available income while they only represent 13.9% of the population. In 2020, a former Uber driver in San Diego sued Uber over alleged racial discrimination in how customer reviews were used to evaluate drivers. Uber relies on a star rating system, which is believed to disproportionately lead to firing people of color or those who speak with accents. In the United States, as of 2020, more than half (55.2%) of Uber drivers are people of color (Uber 2020).

Commentators from both industry and academia have called for action to reduce customer discrimination. In industry, Uber invested more than 10 million dollars in supporting Black-owned small businesses by driving demand to their establishments through promotions and other forms of support. Uber also expanded their supplier diversity program, committing to double spending with Black-owned businesses and contractors.[1] Airbnb initiated Project Lighthouse to uncover, measure and reduce discrimination when booking or hosting on Airbnb.[2]

Law scholars, such as Bartlett and Gulati (2016), have called for legislation against customer discrimination. They argue that we should not take for granted the right of customers to discriminate when exercising their buying power. Indeed, while monitoring and regulating individual customer behavior directly could be controversial and ineffective, especially when personal privacy and autonomy are at stake, Bartlett and Gulati (2016) suggest that the state can use markets to regulate individual customers. Specifically, companies of a certain size "...should have an explicit obligation to curtail and not to facilitate discrimination by their customers."

In this paper, we discuss the *marketing strategy* of a platform in the presence of customer discrimination. When it comes to discriminatory business activity, marketing plays a unique role. As pointed out by Bedi (2019), while many business decisions, such as discrimination against employees, are internal, marketing reaches audiences in a way that internal discriminatory business decisions may not. Marketing is focused on providing services and products that align with customer preferences. A longstanding mantra in the marketing industry goes: "the consumer is king". As opportunities for customer discrimination grow in today's economy, this golden rule is being challenged. Bedi (2019) argues that catering to customer preferences can contribute to "cultural imperialism", where a dominant cultural viewpoint renders other cultural groups invisible. Therefore, in a similar spirit to Bartlett and Gulati (2016), Bedi (2019) argues that marketers may even have a positive obligation to counteract and mitigate this kind of cultural harm. Specifically, companies should pursue culturally inclusive marketing strategies (Bedi 2019). Whereas traditional marketing ethics focuses on the ill effects of advertising harmful products, promoting materialism, and deceptive behavior, the growing challenge of customer discrimination adds a new dimension to marketing ethics. To study this new dimension, we need new theory.

We examine how optimal marketing strategy is impacted by customer discrimination and work towards developing marketing strategies that are inclusive of a diversity of sellers. We are specifically interested in the inclusive allocation of the marketing effort of platforms in promoting sellers. This notion of marketing effort is abstract and can refer to a variety of marketing strategies, including marketing expenditure, advertisement, financial support, promotions, customer service, after-sale support, ancillary services, customer relationship management, sales effort, etc. The

[1] `https://www.uber.com/ca/en/u/right-to-move/`

[2] `https://www.airbnb.ca/resources/hosting-homes/a/a-new-way-were-fighting-discrimination-on-airbnb-201`

million dollars Uber spent on supporting Black-owned small businesses is an example of a marketing effort. Other researchers have used the same abstract notion of marketing effort in related contexts (Li et al. 2020, He et al. 2022).

More specifically, we investigate the following three research questions. First, we evaluate the optimal marketing strategy for an inequality-neutral platform under customer discrimination. For an inequality-neutral platform, its utility is given by the total seller profit (i.e., the size of the pie of transactions on the platform). This goal of maximizing total seller profit is consistent with the platform's own profit maximization when we assume (as is common) that the platform gets a fixed proportion of the value generated in each transition. We ask, how inclusively is marketing effort allocated by an inequality-neutral platform? Next, we evaluate the impact of some external social pressure for more equal market share distribution on the optimal marketing strategy of an inequality-neutral platform. This raises the question: Would a group of protected sellers be better off if the inequality-neutral platform is forced to achieve certain fair market share distribution? Finally, we evaluate the optimal marketing strategy of an inequality-averse platform. We ask how a platform with various degrees of inequality aversion influences the inclusivity of the marketing effort allocation.

We present a parsimonious theoretical model where a two-sided platform wants to allocate marketing effort among its sellers. We use the multinomial logit (MNL) choice model to model customer behavior. We chose the MNL choice model because of its ubiquity in both theory and practice. By analyzing the MNL model, we can understand customer discrimination impacts a standard model for revenue and product-line management. In the MNL choice model, each seller is endowed with a basic utility, which represents its attractiveness to customers. The basic utilities of all sellers determine the probability that any customer would purchase from each seller. This probability also represents the market share of that seller. We assume the existence of customer discrimination. That is, the basic utility of each seller perceived by customers is biased due to customer discrimination. The basic utilities of culturally underrepresented sellers perceived by the customers might be lower than their actual values.

By allocating marketing effort, the platform can increase the basic utilities of sellers through promotional effort. The platform must decide on an allocation plan that distributes finite marketing effort among the different sellers. To investigate our three research questions, we consider three objectives for the platform: total profit maximization, fairness-regulated total profit maximization, and inequality-averse utility maximization. The inequality-averse utility can be naturally represented by the notion of α-fairness (Atkinson 1970) (see Section 5.1 for additional discussion).

We now describe some of our findings. We first show that the optimal inequality-neutral (i.e., total profit maximization) marketing strategy is to allocate all marketing effort to a single seller.

We call this a *concentrated* marketing strategy. We show in examples and discussions that concentrated marketing strategies are prone to discriminatory outcomes because a concentrated marketing strategy tends to further amplify mainstream preferences and diminish the visibility of diverse sellers.

To combat this lack of diversity in marketing effort allocation, a natural idea that has gained cultural relevance is to guarantee a reasonable proportion of market share for a protected group. That is, to ensure that the total market share of a protected group (e.g., a group of culturally underrepresented sellers) to be least β percent of the overall market share. Recall that Cox (2017) shows that underrepresented Airbnb hosts, which represent 86.1% of the population, only earned 26.3% of the total income across all 72 predominantly Black New York City neighborhoods. Therefore, it is reasonable to think that the market would be fairer if the total market shares of underrepresented hosts constitute at least, let's say, 50% of the total market share in those neighborhoods.

To this effect, we analyze the scenario where a platform faces social pressure to reduce inequality by imposing a fair market share constraint. In this scenario, we show that the optimal inequality-neutral (i.e., total profit maximizing) marketing strategy constrained by this fair market share constraint promotes at most two sellers. With such a constraint, the platform may no longer concentrate all its marketing effort on one seller in the culturally dominant group since it can violate the fair market share constraint. Therefore, the inequality-neutral platform spares some marketing effort for one protected seller to satisfy the constraint and maximize total profit. The intuition is straightforward: To promote the total sales of the protected group, the easiest way is to concentrate all spared marketing effort on one representative seller from the protected group and promote the sales of this representative seller. Such practice overlooks every other seller in the protected group and cannot improve the overall fairness of market outcomes. This outcome is reminiscent of the notion "tokenism" in hiring practices (Kanter 1977).

We draw the following insight from this result. An external constraint has a limited effect because strategic decision-makers can always find the cheapest way to satisfy or avoid the constraint. This insight is consistent with research on algorithmic fairness. For example, Fu et al. (2022) analyzes the fairness in algorithmic decision-making. They show that some fairness constraints imposed by law may not be able to deliver some of the anticipated benefits because strategic decision makers (i.e., profit-maximizing firms) would underinvest in learning effort to satisfy the constraint.

In our third result, we analyze the marketing strategy for the platform with inequality aversion. We show that with a certain level of inequality aversion, this strategy may overcome tokenism and result in an inclusive marketing effort allocation among a diverse group of sellers. We show that with enough marketing effort and a high enough level of inequality aversion, the difference between the largest and smallest market shares in the optimal strategy can be arbitrarily small. In this way, we avoid tokenism-type outcomes to get a more inclusive marketing strategy, as Bedi (2019) advocates.

Finally, we provide insights into the impact of different degrees of inequality aversion. If the level of inequality aversion is sufficiently low, the optimal marketing strategy will still be concentrated. This seller who gets most of the marketing effort dominates the market. In contrast, if the level of inequality aversion is sufficiently high, the optimal marketing strategy makes every seller almost totally equal, and we are possibly promoting the seller with low quality because this type of seller gets more marketing effort. Such trade-off reflects the intrinsic conflict between fairness and efficiency, which has been noted elsewhere in different contexts (Bertsimas et al. 2011, 2012). In the context of marketing strategy, the most efficient strategy is neither overly concentrating nor overly inclusive. The "fairest" strategy is diverse and inclusive but not efficient, especially because such a strategy not only compensates sellers for discrimination but may also subsidize sellers with low quality who may not be fit competitors in the marketplace.

Summary of contributions

Our paper makes several contributions. Diversity, equity, and inclusion (DEI) is an emerging topic in the management sciences and has important societal implications. Although the concept of customer discrimination has been studied in economics, law, and other social sciences for some time, to our knowledge, we are the first to introduce this form of discrimination in the management sciences with a formal modeling approach.

Second, we contribute to the discourse on ethics and cultural diversity in marketing. We are the first to propose an analytical framework to analyze the fairness aspect of marketing strategies under customer discrimination. With customer discrimination, catering to customer preferences is not only inappropriate but potentially harmful. In this scenario, the optimal concentrated marketing strategy is exclusive and possibly exacerbates unfair outcomes. We show that the optimal marketing strategy of a platform with high enough level of inequality aversion can achieve more diverse outcomes.

Third, our paper contributes to the analysis of fairness constraints. Researchers have analyzed the effect of various fairness constraints in various scenarios (Fu et al. 2022, Bertsimas et al. 2011, 2012, Li et al. 2019, Bateni et al. 2022, Cohen et al. 2021). Our paper analyzes the effect of one form of fairness constraint when a platform allocates limited marketing effort among independent sellers. Our results show that external constraints can only receive limited effect and may result in tokenism because strategic decision-makers find cheap ways to satisfy these constraints. This insight is consistent with some findings in the extant literature. We also show that imposing a fairness objective—instead of a fairness constraint—offers more "bite" in ensuring inclusive strategies. This result could be of interest to regulators and other social actors who are working to put pressure on platforms to reduce discriminatory outcomes.

Organization of the paper

The paper is organized as follows. The next section contains a literature review. In the main sections of the paper, we consider three regimes under which a platform decides its marketing strategy. The first (Section 3) is when the goal of the platform is to maximize the total profit of all sellers on its platform. We call this the *inequality-neutral* regime. In the next two sections, we look at scenarios where social pressure forces the platform to move away from the total profit maximizing regime. Section 4 explores the scenario where the platform constrains its marketing allocation to ensure that a protected class of sellers gets a minimum proportion of the total market share. Section 5 explores the scenario where the optimal marketing strategy of a platform with inequality aversion preference. Section 6 offers numerical simulations that yield further insight into the structure of optimal marketing effort allocation. Section 7 concludes and points to further research directions.

2. Literature review

This work is inspired by the discourses on customer discrimination in law, economics, and business.

In the law literature, Bartlett and Gulati (2016) summarize common forms of customer discrimination. After refuting two significant rationales for why society might choose to ignore customer discrimination, Bartlett and Gulati (2016) sketch out a policy approach to curtail customer discrimination. While Bartlett and Gulati (2016) discuss customer discrimination in general, Bedi (2019) specifically studies the ethical issues of marketing under customer discrimination. Bedi (2019) challenges the longstanding mantra "customer preferences are king" in the case where customers are discriminatory. Bedi (2019) concludes that marketers have a positive obligation to counteract the cultural harm caused by customer discrimination by promoting cultural diversity in their marketing activities, even if this is against customer preferences. Inspired by these arguments, our work proposes an analytical framework (unexplored in the law literature) to study the diversity of marketing strategies under customer discrimination.

As for formal modeling of discrimination, Becker (1957) was (to our knowledge) the first author to use an economic modeling framework to study discrimination. By analyzing equilibrium outcomes, Becker (1957) argues that greater market competition acts as a strong force to reduce discrimination in the employment process. However, due to discrimination against employees by customers, that competition would not fully eliminate discrimination. Based on his analysis, discriminatory customers would pay a higher price to avoid doing business with underrepresented employees, thus subsidizing discrimination. While Becker (1957) studies discrimination in employment (i.e., wages and hiring process), our work expands the scope of discussion to marketing.

The impact of cultural diversity on business outcomes has been of growing interest in recent years in the business literature. Diversity in the workforce presents a kind of paradox, offering benefits such as enhanced innovation, productivity, and profitability (Zhang 2020, Nathan and Lee 2013, Phillips et al. 2009, Carter et al. 2010) while at the same time potentially leading to challenges such as reduced team cohesion and increased difficulties in communication (Hamilton et al. 2012, Towry 2003). A similar controversial effect of diversity is also found in marketing. Brands that aim to establish a distinct social identity, particularly in the fashion industry, may experience a decline in demand if individuals outside of their target demographic begin to adopt them (Douglas and Isherwood 2021, Berger and Heath 2007). This is due to the dilution of the intended signal or message associated with the brand's social identity (Berger and Heath 2007). For example, White consumers may refrain from engaging in certain consumption behaviors and practices, as they aim to avoid conveying a negative social identity to their co-ethnic group that may be associated with the product (Berger 2008, Cooper and Jones 1969); studies have revealed that the inclusion of minorities in advertising can result in a decreased purchase intention among non-underrepresented consumers (Kerin 1979, Whittler and DiMeo 1991, Grier and Deshpandé 2001, Aaker et al. 2000). This fear may drive more White imagery in marketing (Puntoni et al. 2011). On the other hand, empirical evidence in the marketing industry emphasizes the significance of diversity. Research conducted by Adobe shows that 61% of U.S.-based consumers believe that diversity in advertising is important, and 38% are more likely to trust brands that show diversity in their ads.[3]

From a methodological perspective, our work builds on an analytical framework that is common in revenue management and product line design. Our foundation is the multinomial logit (MNL) choice model, commonly used in research and industrial practice (Li and Huh 2011, Rusmevichientong et al. 2014, Feldman et al. 2022, Dong et al. 2009, Li et al. 2020, He et al. 2022). Researchers in revenue management use the MNL model to develop revenue-maximization strategies such as pricing, assortment, recommendation, inventory control, and product line design and marketing. It is also applied to analyze market equilibrium (Gallego et al. 2006). Empirical researchers also use the MNL model to estimate demand, substitute, and loss of sales (Vulcano et al. 2012, Berbeglia et al. 2022). To our knowledge, we are the first to use the MNL model to understand the impact of marketing interventions on discriminatory behavior. A key modeling feature of our framework is the "marketing effort" decision variable in a multinomial logit (MNL) framework, an additional term added to the attraction value of the MNL model that captures promotional effects by the platform. That is, marketing effort increases the attraction value of the promoted seller. He et al. (2022) propose a similar form of "marketing effort" to analyze the product line marketing with the MNL model. They show that the revenue-maximizing marketing strategy concentrates all marketing effort on one flagship product under a convex cost function. Their work explains the existence of

[3] https://www.slideshare.net/adobe/adobe-digital-insights-diversity-in-advertising-2019

concentrated marketing effort allocation strategies and flagship products. Our work analyzes marketing strategy from a different perspective. We discuss the implication of concentrated marketing effort under customer discrimination and introduce a new dimension of diversity and inclusion.

Li et al. (2020) study the joint optimization of product quality and prices problem under the MNL choice model. In their work, the attraction value of the MNL model is the sum of individual terms representing quality, prices, and an exogenous constant. They jointly optimize the qualities and prices to maximize the total revenue in the MNL model. Their notion of "quality" captures the products' attributes that can be added or deleted by the firm, while we consider the marketing effort can be allocated to this independent seller by a platform. For instance, ancillary services such as a spa in a resort hotel are examples of "quality", while coupons provided by Uber belong to "marketing effort".

One key ingredient in our work is the notion of fairness. There is a large body of discussion about the notion of fairness (Wong et al. 1982, Jain et al. 1984, Dianati et al. 2005, Koksal et al. 2000, Bredel and Fidler 2009, Mo and Walrand 2000, Kelly et al. 1998). We refer readers to Young (1995) for detailed exposition. These notions range from simple ratios between the largest and smallest utility to more complicated measures such as entropy. Our work focuses on the α-fairness scheme, which was studied early on by Atkinson (1970). The parameter α is viewed as a fairness measure in the sense that a larger α represents a stronger desire for fairness (Uchida and Kurose 2009, Bonald and Massoulié 2001, Massoulie and Roberts 1999). Lan et al. (2010) develop an axiomatic approach to analyze α-fairness and provide an interpretation of "larger α is more fair". Bertsimas et al. (2012) characterize the trade-off between efficiency and fairness for α-fairness scheme. Built on these discussions, our work explores the practical effects of α-fairness in the setting of marketing under customer discrimination.

The aforementioned notions of fairness are widely applied in the fair resources allocation problem. In this problem, a central planner needs to allocate certain limited resources to a number of distinct entities. The planner knows those entities' preferences, which are described via cardinal utilities. The central problem is then concerned with how the central planner should allocate these resources with some fairness consideration. For instance, Bertsimas et al. (2011, 2012) formulate a problem where a decision maker allocates utilities in a feasible set to maximize α-fairness. Inspired by α-fairness metric, Li et al. (2019) propose a similar fairness objective, q-fairness, to encourage a more fair accuracy distribution across devices in the context of federated learning. Bateni et al. (2022) study the setting that a platform dynamically allocates a collection of goods to buyers in an online fashion by maximizing weighted proportional fairness. More generally, fairness has been considered in the context of fair operations, including algorithmic fairness (Fu et al. 2022, Kallus et al. 2022), healthcare (Bertsimas et al. 2013), transportation (Barnhart et al. 2012), network system (Altman et al. 2008), portfolio optimization (Iancu and Trichakis 2014), revenue management (Cohen et al. 2021). Our work proposes an analytical framework for the fair allocation of limited marketing effort, making a notable contribution to the literature on fair resource allocation and fair operations.

3. Inequality-neutral regime

In this section, we consider the problem of an inequality-neutral platform allocating marketing effort to sellers in order to maximize the total profit of all sellers. Section 3.1 sets up a model of this problem, which is analyzed in Section 3.2. Section 3.3 considers the problem when the platform chooses both marketing effort allocations and prices of the offerings. Section 3.4 offers further interpretations of the results and delineates managerial insights.

3.1. Model setup

Consider a two-sided platform that hosts N sellers. By convention, we let $[N]$ denote the set of sellers. We assume customer preferences follow a standard multinomial logit (MNL) customer choice model.[4] That is, the market share s_i of the i-th seller is $s_i \doteq e^{u_i}/(1+\sum_{j=1}^N e^{u_j})$, where u_i represents the basic utility of the product or service offered by seller i. There is an outside option where customers select none of the sellers on the platform. For simplicity, we will say "offering i" to refer to the product or service offered by seller i. We call the exponential term e^{u_i} the attraction value of offering i. The marginal profit of seller i is r_i. Thus, the profit of seller i is $\pi_i \doteq r_i s_i$, and the total profit of all sellers is $\pi \doteq \sum_{i\in[N]} \pi_i$. To avoid some degenerate cases, we assume that there do not exist two different sellers $i, j \in [N]$ such that $r_i = r_j$ and $u_i = u_j$.

We consider the scenario that the platform allocates a finite amount of marketing effort among all sellers. Let x_i denote the marketing effort allocated to seller i. We call $\mathbf{x} = (x_1, \ldots, x_N)$ the marketing strategy. Given the allocated marketing effort x_i, the total utility of offering i becomes $u_i + x_i$. As a result, the market share and profit of seller i becomes

$$s_i(\mathbf{x}) \doteq \frac{e^{u_i+x_i}}{1+\sum_{j\in[N]} e^{u_j+x_j}} \quad \text{and} \quad \pi_i(\mathbf{x}) \doteq r_i s_i(\mathbf{x}), \tag{1}$$

respectively. Note that $s_i = s_i(\mathbf{0})$.

Let $\pi(\mathbf{x}) \doteq \sum_{i\in[N]} \pi_i(\mathbf{x})$ be the total profit of all sellers for marketing strategy $\mathbf{x}$. With marketing effort x_i, the total attraction value of offering i becomes $e^{u_i+x_i}$. The increment of attraction value is given by $(e^{x_i}-1)e^{u_i}$. We call $y_i(\mathbf{x}) \doteq e^{x_i} - 1$ the effect of marketing effort. In particular, if the allocated marketing effort x_i is 0, the effect y_i is also 0. For now, we will assume that the platform cannot choose prices, and we take them as exogenous. This allows us to isolate our attention on the impact of marketing effort. We will relax this assumption in the next section.

[4] For a reader who is possibly unfamiliar with the standard multinomial logit choice model, we recommend the popular reference (Train 2009).

Our notion of marketing effort captures the positive effect of general marketing campaigns rather than modeling specific marketing strategies. This allows us to encompass a range of marketing practices. For example, a common challenge faced by Lyft is known as the "incentive allocation problem". In order to maximize incremental rides or driving hours, Lyft offers promotional coupons to riders and bonuses to drivers.[5] These promotional coupons function as indirect price changes, which can influence the purchasing probability of customers. In the MNL customer choice model, this effect is modeled by adding the term x_i to the basic utility u_i of offering i. As a result, the total attraction value of offering i becomes $e^{u_i+x_i}$.

Naturally, the total effect of marketing effort is limited due to various reasons such as a limited financial budget, the limited influence of marketing campaigns, or the presence of some cost function for marketing effort (e.g., He et al. (2022)). He et al. (2022) show that with a strictly convex cost function, the total effect $\sum_{i\in[N]} y_i(\mathbf{x}^*)$ of optimal marketing strategy $\mathbf{x}^*$ is finite. Furthermore, they show that given a fixed amount of total effect $\sum_{i\in[N]} y_i$, the corresponding (generically unique) profit-maximizing marketing strategy $\mathbf{x}^*$ is concentrated in the sense that there exists one $i\in[N]$, which is referred to as "flagship", such that $x_i^*>0$ and $x_j^*=0$, for all $j\neq i$. In this paper, we adopt the terminology used in He et al. (2022) and refer to a marketing strategy $\mathbf{x}$ as concentrated on seller $i\in[N]$ if $x_i>0$ and $x_j=0$, for all $j\neq i$. Instead of a specific form of a cost function, we introduce the following budget constraint on the marketing effort

$$\sum_{i\in[N]} y_i(\mathbf{x}) \le M, \tag{2}$$

where M is the total budget of marketing effect. Note that the budget constraint (2) in our model could be a result of imposing a cost function, as in He et al. (2022).

In this paper, we consider the problem that the platform maximizes certain objectives (i.e., total profit or certain notions of fairness) by deciding a marketing strategy $\mathbf{x}\doteq(x_i)_{i\in[N]}$ subject to the budget constraint (2). Our goal is to investigate the optimal distribution of marketing budget among sellers under varying criteria of fairness. In this regard, He et al. (2022) show that in a multi-product production line, the profit-maximizing marketing strategy is to concentrate all effort on one "flagship" product. However, in contrast to this concentrating result, we are interested in exploring if the marketing budget could be distributed more evenly among sellers and, consequently, if the optimal strategy could result in a more even distribution of market outcomes, including market shares and profits. These questions are particularly relevant under the presence of customer discrimination. As we will show in this work, in a discriminatory environment, a more even distribution of marketing effort is more desired by sellers. The reader may notice that we do not explicitly quantify the effect of customer discrimination in our customer choice model. Indeed,

[5] `https://eng.lyft.com/how-to-solve-a-linear-optimization-problem-on-incentive-allocation-5a8fb5d04db1`

it is tempting to first quantify the effect of discrimination (say as a term in the MNL choice model) and then allocate marketing effort correspondingly. However, complications arise in this approach. Although it is easy to identify disparity in market outcomes between underrepresented sellers and culturally dominant sellers, it is hard to identify discrimination as the cause of this disparity, not to mention accurately quantify the effect of discrimination. For a detailed discussion of the empirical challenges this raises, see Lang and Kahn-Lang Spitzer (2020). To our best knowledge, there is no solid universal approach to identifying discrimination in the literature. Many researchers turned to experimental evidence to identify discrimination. However, most of the work focused on specific settings and cannot be easily generalized. To avoid getting stuck in this controversial debate, we instead focus on analyzing the actual impact of different marketing strategies. Even if we cannot identify discrimination as the root of unequal outcomes, we propose policies that nonetheless help protect the economic interests of underrepresented sellers.

3.2. Analysis of the inequality-neutral marketing strategy

Our analysis begins with the inequality-neutral (i.e., total profit maximizing) marketing strategy. Suppose the platform maximizes the total profit $\pi(\mathbf{x})$ of all sellers by allocating the limited marketing effort among the sellers. This objective is equivalent to maximizing the platform's revenue if the platform collects a fixed fraction of each seller's profit. Recall that $y_i(\mathbf{x}) \doteq e^{x_i} - 1$ represents the multiplicative factor. Then, the platform solves the following problem:

Problem 1 (Total profit maximization). Choose marketing strategy $\mathbf{x}$ to solve

$$\begin{aligned} \max_{\mathbf{x}\in\mathbb{R}^N} \quad & \sum_{i\in[N]} r_i \frac{e^{u_i+x_i}}{1+\sum_{j\in[N]} e^{u_j+x_j}} \\ \text{s.t.} \quad & \sum_{i\in[N]} y_i(\mathbf{x}) \le M, \\ & x_i \ge 0, \quad \text{for all } i\in[N]. \end{aligned}$$

Notice that prices are exogenous and not decisions of the platform in Problem 1. Firstly, we transform Problem 1 into a linear program from which we can see that the optimal marketing strategy is concentrated in the sense that one seller gets all the marketing effort. We transform the decision variables to the vector $\mathbf{q}$ of market shares from marketing strategy $\mathbf{x}$ by considering the following unique inverse marketing effort function:

$$x_i(\mathbf{q}) = -u_i + \log q_i - \log(1 - \sum_{j=1}^{N} q_j). \tag{3}$$

Next, we substitute equation (3) into Problem 1. The budget constraint becomes

$$\sum_{i\in P} e^{-u_i} q_i \le (1 - \sum_{i\in[N]} q_i)(M+N),$$

which is linear in $\mathbf{q}$. Similarly, the non-negative marketing effort constraint $x_i \ge 0$ becomes the following linear constraint

$$q_i \ge e^{u_i}(1 - \sum_{i\in[N]} q_i), \quad \forall i\in[N].$$

Then, we obtain the following linear version of Problem 1.

PROBLEM 2. Choose market shares vector $\mathbf{q}$ to solve

$$\begin{aligned} \max_{\mathbf{q}\in\mathbb{R}^N} \quad & \sum_{i\in[N]} r_i q_i \\ \text{s.t.} \quad & \sum_{i\in[N]} e^{-u_i} q_i \leq (1-\sum_{i\in[N]} q_i)(M+N), \\ & q_i \geq e^{u_i}(1-\sum_{j\in[N]} q_j), \quad \forall i\in[N], \\ & \sum_{i\in[N]} q_i \leq 1, \\ & q_i \geq 0, \quad \forall i \in [N]. \end{aligned}$$

Since Problem 2 is a linear program, it admits an optimal basic feasible solution $\mathbf{q}^*$. We show that the marketing strategy corresponding to this extreme point solution is concentrated. Note that $\sum_{i\in[N]} q_i^* \leq 1$ cannot be tight at the optimal solution or the budget constraint is violated. Similarly, $q_i^* \geq 0$ cannot be tight for any $i \in [N]$ or the non-negative marketing effort constraint is violated. The budget constraint must be tight at the optimal solution. Therefore, at least $N-1$ of the non-negativity marketing effort constraints and equality constraints are tight, which implies there are exactly $N-1$ sellers in total that get 0 marketing effort, i.e., $x_i^* = 0$. This further implies there is only one seller i^* who gets all marketing effort, i.e., $x_{i^*}^* = \ln(M+1)$.

He et al. (2022) first proved an equivalent concentration result, which is phrased in our setting in Proposition 1 below, by formulating the objective function as a fractional of two linear functions and then utilizing some property of fractional programming. In fact, they rediscover the established result that linear-fractional programming admits an optimal basic feasible solution. Instead, by transforming the problem into a linear program and exploiting the special structures of the constraints in this setting, we give a more concise alternative proof based on the extreme point solutions in linear programming.

PROPOSITION 1 **(Concentrated marketing effort strategy)**. *The generically unique optimal solution $\mathbf{x}^*$ to Problem 1 is concentrated in the sense that $x_{i^*}^* = \ln(M+1)$ for some seller $i^* \in [N]$ and $x_i^* = 0$ for $i \neq i^*$. Specifically, $i^* \in \arg\max_{i\in P} R_i$, where*

$$R_i \doteq \frac{M r_i e^{u_i} + \sum_{j=1}^N r_j e^{u_j}}{M e^{u_i} + 1 + \sum_{j=1}^N e^{u_j}}$$

is the total profit if we concentrate all marketing effort on seller i. We call seller i^* the optimal seller. That is, Problem 1 can be solved by finding the seller with the largest R_i for $i \in [N]$ and concentrating all effort on that seller.

Given marketing budget M, let $\mathbf{x}^i(M)$ be the marketing strategy concentrating on seller i, that is $x^i(M)_i = \ln(M+1)$ for seller i and $x^i(M)_j = 0$ for all $i \neq j$. This result tells us that the optimal marketing strategy $\mathbf{x}^*(M)$ is one of $\{\mathbf{x}^i(M) : i \in [N]\}$ for all budgets M.

Remark 1 (Generically unique optimal solutions). In general, there might be many optimal solutions to Problem 2. For example, suppose all sellers in $[N]$ are identical (something we rule out in our setup, but consider here for the purpose of illustration). Then, concentrating all effort on anyone is optimal. Moreover, the convex combination of two optimal solutions to Problem 2 is also optimal and not necessarily concentrated. However, with a little perturbation in the marginal profits vector $\mathbf{r}$, there will be a unique optimal basic feasible solution.

3.2.1. Characterization of optimal solutions Although Problem 1 can be solved efficiently by either linear program or Proposition 1, we seek further analytical structure, in particular, qualifications for a seller to the optimal. As we will discuss in Problem 1 below, this analytical structure offers insight into the discriminatory nature of the profit-maximizing solution. Naturally, the identity of the optimal seller depends on the interaction between the intrinsic features (i.e., $\mathbf{u}$ and $\mathbf{r}$) and the available budget M. We denote this relationship by $i^*(\mathbf{u},\mathbf{r},M)$. He et al. (2022) analyze two extreme cases. When $M \to \infty$, we have $i^*(\mathbf{u},\mathbf{r},M) = \arg\max_{i\in[N]} r_i$, i.e., only the profitability matters. When the budget is limited, i.e., when M is small, He et al. (2022) shows that the basic utility u_i also influences the identity of $i^*(\mathbf{u},,r,M)$. Our goal is to derive further insights into the relation $i^*(\mathbf{u},\mathbf{r},M)$ and show how these insights relate to customer discrimination.

Our analysis of the relation $i^*(\mathbf{u},\mathbf{r},M)$ proceeds in two steps. First, we reveal a structure for $i^*(\mathbf{u},\mathbf{r},M)$ that is independent of M. Indeed, we derive a set (called the *candidate set*) that depends only on $\mathbf{u}$ and $\mathbf{r}$ that contains $i^*(\mathbf{u},\mathbf{r},M)$ for all budgets M. Second, we provide analytical conditions for when a given element of the candidate set is the optimal seller $i^*(\mathbf{u},\mathbf{r},M)$, as a function of M. Throughout our discussion, we take $\mathbf{u}$ and $\mathbf{r}$ as given and so we drop them from the argument of $i^*(\mathbf{u},\mathbf{r},M)$. In other words, our work is to understand how $i^*(M)$ depends on M for given $\mathbf{u}$ and $\mathbf{r}$.

Let's begin by showing that the marginal profit $r_{i^*(M)}$ of the optimal seller must be larger than the total profit π that arises when the marketing budget is 0; that is, when the platform does not undertake any marketing strategy (see Lemma A.1 in the appendix). Note that the total profit $\pi = \sum_{i\in[N]} \pi_i$ without any marketing strategy is the average marginal profit of all sellers. If the marginal profit r_j of seller j is below the average (i.e., $r_j < \pi$), allocating marketing effort to seller j can only hurt the total profit by cannibalizing demand for a more profitable seller. Therefore, a necessary condition for seller i to be optimal is for $r_i > \pi$.

Second, we show that if seller j is "dominated" (in the sense of Definition 1 below), then seller j is not optimal for any available budget $M > 0$ (see Lemma A.2 in the appendix).

Definition 1 (Dominance relation and the set of competitive sellers). For seller i and seller j such that $r_i > \pi$ and $r_j > \pi$, we say seller i *dominates* seller j if $r_i \geq r_j$ and $(r_i - \pi)s_i \geq (r_j - \pi)s_j$, where s_i is defined after (1). This notion allows us to define the set of *competitive sellers*

$$\mathcal{C} \doteq \{i \in [N] : r_i > \pi \text{ and seller } i \text{ not dominated by any other sellers}\}.$$

Observe that the set $\mathcal{C}$ contains the sellers with the $|\mathcal{C}|$ highest marginal profits given $\mathbf{u}$ and $\mathbf{r}$ (indeed, elements outside of this set all have $r_i \leq \pi$ whereas elements in this set have $r_i > \pi$). We relabel all of the sellers so that the sellers with index $i = 1, 2, \ldots, |\mathcal{C}|$ are the competitive sellers with their marginal profits in decreasing order. That is, $r_1 > \ldots > r_{|\mathcal{C}|}$. The relabeling of the remaining sellers is arbitrary. Recall that at the beginning of Section 3.1, we assume there do not exist two identical sellers in the sense that $r_i = r_j$ and $u_i = u_j$. Thus, for two competitive sellers i, j, it is impossible that $r_i = r_j$, which allows us to construct a strict ordering of the r_i of the competitive sellers.

Since we have already shown that the optimal seller $i^*(M)$ satisfies $r_{i^*(M)} > \pi$, we only define dominance relation for all seller i with $r_i > \pi$. Notice that if $r_i > r_j > \pi$ and $u_i > u_j$, then seller i dominates seller j. However, given $r_i > r_j$, this condition $u_i > u_j$ is not a necessary condition for seller i to dominate seller j; it is possible that even when $u_i < u_j$ holds (in turn, $s_i < s_j$), seller i dominates seller j as long as $(r_i - \pi)s_i > (r_j - \pi)s_j$ still holds. That is, a seller with a larger marginal profit r_i and slightly smaller basic utility u_i could still dominate. Hence, our definition of dominance relation is weaker than $r_i > r_j$ and $u_i > u_j$. Moreover, since we label competitive sellers in descending order of their marginal profits, it follows that the basic utilities of competitive sellers are in ascending order, i.e., $u_1 < \ldots < u_m$, or equivalently, $s_1 < \ldots < s_m$ (see Lemma A.3 in the appendix).

We have shown that one necessary condition for i to be an optimal seller is that i is a competitive seller, i.e., $i \in \mathcal{C}$. However, a competitive seller may not be optimal for any $M > 0$. This happens when this seller is "jointly dominated" in the following sense.

Definition 2 (Joint dominance relation and candidate sellers). A seller $j \in \mathcal{C}$ is *jointly dominated* by sellers $i, k \in \mathcal{C}$ if $i < j < k$ and

$$r_j s_j < r_i s_i \frac{q_k - q_j}{q_k - q_i} + r_k q_k \left(1 - \frac{q_k - q_j}{q_k - q_i}\right).$$

The set of *candidate sellers* is

$$\mathcal{D} \doteq \{i \in \mathcal{C} : \text{seller } i \text{ is not jointly dominated by any pair of competitive sellers}\}.$$

As before, we relabel the sellers in $\mathcal{C}$ so that the candidate sellers in $\mathcal{D}$ are labeled in descending order of marginal profits, i.e., $r_1 \geq \ldots \geq r_{|\mathcal{D}|}$. The remaining sellers in $\mathcal{C}$ are ordered arbitrarily.

Example EC.1 in the appendix provides a concrete example of joint dominance. The intuition behind the joint dominance notion is that while seller j can be better than seller i for some M and better than seller k for some other value of M, for any given value of M, it cannot outperform both seller j and k. Note that $\mathcal{D}$ is not empty since given any $M > 0$, the corresponding optimal seller $i^*(M)$ must be in $\mathcal{D}$ since it outperforms all other sellers at the budget level M.

The candidate set notion gives rise to a precise qualification for a seller to be optimal for a given choice of u, r, and M. In Theorem 1, we show that a seller i is a candidate seller (i.e., $i \in \mathcal{D}$ if and

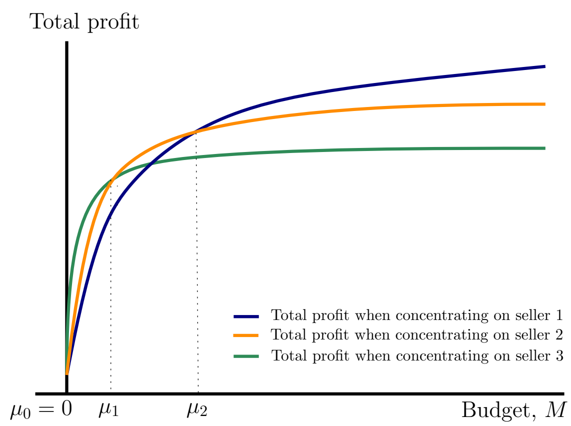


Figure 1 This figure shows the total profit of marketing strategies that concentrate on sellers 1,2, and 3, respectively. As budget M increases from 0 to infinity, the optimal strategy concentrates on seller 3, then seller 2, and finally seller 1.

only if there exists some $M > 0$ such that seller i is optimal. That is to say, the marginal profits $\mathbf{r}$ and basic utilities $\mathbf{u}$ decide $i^*(M)$ (up to the selection of M) by determining the candidate sellers through the dominance and joint dominance relations. Moreover, we show that among candidate sellers, as the available budget M increases from 0 to ∞, candidate sellers become optimal in turn following the order from $|\mathcal{D}|$ to 1.

THEOREM 1. *Let $\mathbf{u}$ and $\mathbf{r}$ be given. Seller i is optimal for $M > 0$ if and only if seller $i \in \mathcal{D}$. Moreover, given the ordering of candidate sellers $i = 1, \ldots, |\mathcal{D}|$, there exists a sequence of thresholds $\{\mu_k\}_{0 \le k \le |\mathcal{D}|}$ with $\infty = \mu_0 > \mu_1 > \ldots > \mu_{|\mathcal{D}|-1} > \mu_{|\mathcal{D}|} = 0$ such that seller i is optimal when $M \in (\mu_i, \mu_{i-1})$.*

According to the first part of Theorem 1, for every seller $i \in \mathcal{D}$, there exists a budget value M such that seller i is optimal. On the other hand, sellers not in $\mathcal{D}$ will never receive any marketing effort regardless of the budget available. The second part of Theorem 1 demonstrates that the platform must balance between promoting the most profitable seller and promoting the most intrinsically popular seller (i.e., the seller with the largest basic utility u_i) to maximize the total profit. As the available budget M increases from 0 to ∞, marketing effort allocations shifts among candidate sellers from the most intrinsically popular (that is, with the largest u_i) to the most profitable (that is, with the largest r_i). No such result can be found in the literature. The following example shows the transfer of optimal marketing strategy as M increases from 0 to infinity.

EXAMPLE 1 (CANDIDATE SELLERS BECOME OPTIMAL IN TURN). Suppose there are 3 sellers with marginal profits vector $(r_1, r_2, r_3) = (5.7, 5.6, 5.5)$ and basic utility vector $(u_1, u_2, u_3) = (0.1, 1, 2)$. All sellers are candidate sellers. As budget M increases from 0 to infinity, the optimal seller shifts from $i = 3$ to $i = 1$.

In certain industries, sellers may have comparable marginal profits (e.g., horizontally differentiable markets), or the platform may lack information about the profitability of each seller. Furthermore, researchers have shown a positive correlation between market share and ROI (i.e., return of investment) (Buzzell et al. 1975). In these cases, the platform may prioritize maximizing the total market share rather than the total profit. To analyze these settings, we specifically consider the scenario where all sellers are equally profitable, i.e., $r_i \equiv r > 0$ for all $i \in [N]$. In this scenario, maximizing the total profit is equivalent to maximizing the total market share. In particular, if $r = 1$, the total profit equals the total market share. In this case, we show the following corollary.

COROLLARY 1 **(Market-share maximization marketing strategy)**. *The optimal market share maximization strategy (i.e., optimal solution to Problem 1 with $r_i \equiv r > 0$) is to concentrate all marketing effort on the seller $i \in [N]$ with the highest basic utility u_i.*

Results in this section demonstrate that the profit-maximizing marketing strategy is concentrated, i.e., the platform exclusively places all of its marketing effort on a single product. However, such exclusive marketing practices can result in discriminatory outcomes in the context described in the following example.

EXAMPLE 2 (IPOD SELLING ON A ONLINE MARKET (DOLEAC AND STEIN 2013)). In a field experiment on an online market, Doleac and Stein (2013) find that iPods held by dark-skinned sellers received 18% fewer offers than iPods held by light-skinned sellers. Moreover, a dark-skinned seller's average offer is approximately $5.72 lower than a light-skinned seller's, with an even greater difference in the highest offers: the best offer received by a dark-skinned seller is typically $7.07 lower. Reflecting these facts in a customer choice model, suppose that due to customer discrimination, the attraction values of dark-skinned sellers are lower than the attraction values of light-skinned sellers, and dark-skinned sellers have lower profit margins due to discrimination. Then, if the platform allocates marketing effort to maximize the total profit or total sales quantity, the optimal marketing strategy is to only promote light-skinned sellers. In this case, implementing the optimal marketing strategy, the platform actually facilitates customer discrimination. This results in an even larger racial disparity in market outcomes. Such a marketing strategy violates the principle proposed in Bartlett and Gulati (2016), which states: "entities should have an explicit obligation to curtail and not to facilitate discrimination by their customers".

Furthermore, we can formalize how a concentrated strategy can be harmful because it can reduce market shares for non-optimal sellers, as we demonstrated in the above example. For the general case, we can quantify the loss suffered by sellers who do not get any marketing effort under the profit-maximizing strategy. Suppose seller i^* gets all marketing effort under the profit-maximizing strategy. Then, the loss in market share of seller $i \neq i^*$ is

$$L_i(M) \doteq s_i - s_i(\mathbf{x}^*(M)) = s_i \frac{Me^{u_{i^*}}}{1 + \sum_{j=1}^N e^{u_j} + Me^{u_{i^*}}} \in [0, s_i), \tag{4}$$

where $s_i(\mathbf{x})$ and s_i are defined in (1). The loss $L_i(M)$ is increasing from 0 to s_i as the total marketing budget M increases from 0 to infinity. That is, as the platform puts in more effort, seller i will eventually lose all of its market share.

3.3. Analysis of the optimal joint pricing and marketing strategy

The analysis in Section 3.2 does not explicitly examine the effect of prices. Instead, the impact of prices is incorporated into the utility and marginal profit. With arbitrarily given prices, we can analyze the scenario where prices are externally determined and not set by the platform. In this subsection, we analyzed a different case in which the prices are also set by the platform. For instance, Airbnb uses a pricing algorithm to suggest prices for hosts, with many hosts adopting the suggested prices. (Zhang et al. 2021). We investigate structural properties associated with the optimal pricing decision.

We further decompose the utility value as $u_i = a_i - b_i p_i$, where a_i represents the offering's intrinsic quality, b_i represents the price sensitivity parameter, and p_i is the price of the offering. Similarly, we decompose the marginal profit as $r_i = p_i - c_i$, where c_i is the marginal cost. Such a model with joint pricing and marketing decision is not found in He et al. (2022).

The next assumption assures that sellers are not identical. We assume this to avoid trivial cases where multiple sellers are not distinguishable, leading to multiple optimal solutions.

ASSUMPTION 1 **(Heterogeneous sellers)**. *Assume sellers are not identical in the sense that* $a_i - b_i c_i \neq a_j - b_j c_j$ *holds for all* $i \neq j$.

Consider the problem that the platform maximizes its total profit by deciding prices and allocating marketing effort. Recall that $y_i(\mathbf{x}) \doteq e^{x_i} - 1$ represents the multiplicative factor.

PROBLEM 3 (JOINT PRICING AND MARKETING PROFIT-MAXIMIZING PROBLEM). Choose price vector $\mathbf{p}$ and marketing strategy $\mathbf{x}$ to solve

$$\begin{aligned} \max_{\mathbf{p},\mathbf{x}\in\mathbb{R}^N} \quad & \frac{\sum_{i\in[N]}(p_i - c_i)e^{a_i - b_i p_i + x_i}}{1 + \sum_{j\in[N]} e^{a_j - b_j p_j + x_j}} \\ \text{s.t.} \quad & \sum_{i\in[N]} y_i(\mathbf{x}) \leq M, \\ & \mathbf{x}, \mathbf{p} \geq 0. \end{aligned}$$

Problem 3 cannot be transformed into a concave maximization problem by applying the inverse price function. To see this, consider the following inverse prices function similar to Section 3.2:

$$p_i(\mathbf{q}, \mathbf{x}) = \frac{1}{b_i}(a_i + x_i + \log(1 - \sum_{j=1}^{N} q_j) - \log q_i), \quad \forall i \in [N].$$

Then, the objective becomes a function of market share vector $\mathbf{q}$ and marketing strategy $\mathbf{x}$

$$\sum_{i\in[N]} (p_i(\mathbf{q}, \mathbf{x}) - c_i) q_i,$$

which is not necessarily concave. For instance, in the case $N=1$, this is neither concave nor convex. Nevertheless, we show that Problem 3 can be solved efficiently, and we characterize the structure of optimal solutions as follows.

Let $\pi(\mathbf{p},\mathbf{x})$ be the total profit given pricing strategy $\mathbf{p}$ and marketing strategy $\mathbf{x}$. If we fix marketing strategy $\mathbf{x}(M)$ and choose prices $\mathbf{p}$ to maximize the total profit $\pi(\mathbf{p},\mathbf{x}(M))$, then the optimal prices vector is unique (see, e.g., Li and Huh (2011)). Define this unique solution as $\mathbf{p}(\mathbf{x}(M)) \doteq \arg\max_{\mathbf{p}} \pi(\mathbf{p},\mathbf{x}(M))$. Moreover, (Li and Huh 2011) show that $\mathbf{p}(\mathbf{x}(M))$ can be easily computed by a bisection search algorithm for any given $\mathbf{x}(M)$. On the other hand, if we fix prices $\mathbf{p}$ and choose marketing strategy $\mathbf{x}(M)$ to maximize the total profit $\pi(\mathbf{p},\mathbf{x}(M))$, then the optimal marketing strategy is one of $\{\mathbf{x}^i(M)\}_{i\in[N]}$ characterized by Theorem 1. Therefore, to solve the non-convex Problem 3, we only need to check at most N pairs of solutions $(\mathbf{p}(\mathbf{x}^i(M)),\mathbf{x}^i(M))$, and each pair can be computed efficiently. We prove this statement formally in Proposition 2.

PROPOSITION 2 **(Optimal solution to Problem 3)**. *Let $(\mathbf{p}^*(M),\mathbf{x}^*(M))$ be an optimal solution to Problem 3. For any $M\geq 0$, $\pi(\mathbf{p}^*(M),\mathbf{x}^*(M)) = \max_{i\in[N]}\pi(\mathbf{p}(\mathbf{x}^i(M)),\mathbf{x}^i(M))$. Moreover, $\pi(\mathbf{p}(\mathbf{x}^i(M)),\mathbf{x}^i(M))$ is the unique solution for ρ in the following expression*

$$\rho = \sum_{k\in[N]} \frac{e^{a_k-b_kc_k-1}}{b_k} e^{-b_k\rho} + M\frac{e^{a_i-b_ic_i-1}}{b_i} e^{-b_i\rho}.$$

After solving this equation to find $\pi^i := \pi(\mathbf{p}(\mathbf{x}^i(M)),\mathbf{x}^i(M))$, we can compute the price vector $\mathbf{p}(\mathbf{x}^i(M))$ for any $i\in[N]$ as $\mathbf{p}(\mathbf{x}^i(M))_k = c_k + 1/b_k + \pi^i$ for all $k\in[N]$.

Next, similar to Section 3.2, we analyze the qualification of seller i to guarantee that the corresponding pair $(\mathbf{p}(\mathbf{x}^i(M)),\mathbf{x}^i(M))$ is optimal. We have one extra layer of complexity due to the pricing decision. Again, we can show that if a seller j is "dominated" by some other seller, then the corresponding pair $(\mathbf{p}(\mathbf{x}^i(M)),\mathbf{x}^i(M))$ is not optimal for any $M>0$ (see Lemma A.10 in the appendix). The precise notion of "dominance" is as follows.

DEFINITION 3 (DOMINANCE RELATION AND COMPETITIVE SELLERS). For seller i and seller j, we call seller i dominates seller j if $b_i\leq b_j$ and $a_i-b_ic_i-\ln b_i-b_i\pi_0 \geq a_j-b_jc_j-\ln b_j-b_j\pi_0$. Define the set of *competitive sellers* as

$$\mathcal{C}_2 \doteq \{i\in[N]: \text{ seller } i \text{ not dominated by any other sellers}\}.$$

Similar to before, we relabel all of the sellers so that the competitive sellers are labeled first and in ascending order of their price sensitivity parameters, i.e., $b_1<\ldots<b_{|\mathcal{C}_2|}$. Notice that $b_i\neq b_j$ for any $i,j\in\mathcal{C}_2$. To see this, suppose $a_i-b_ic_i<a_j-b_jc_j$ (by Assumption 1, $a_i-b_ic_i=a_j-b_jc_j$ is not possible) and $b_i=b_j$. Then, seller j dominates seller i. This contradicts the fact that they are competitive sellers.

We make the following observations. First, notice that sellers with larger price sensitivity parameters cannot dominate sellers with smaller price sensitivity parameters. Second, in the case of identical price sensitivity parameters (i.e., $b_i \equiv b$ for all $i \in [N]$), the seller with the largest $a_i - b_i c_i$ dominates all the other sellers. In this case, this seller is only one competitive seller. Next, in the case that $b_i < b_j$, seller i dominates seller j as long as $a_j - b_j c_j - \ln b_j$ is not significantly larger than $a_i - b_i c_i - \ln b_i$, i.e., the difference is bounded by $(b_j - b_i)\pi_0$, where π_0 is the optimal total profit when $M = 0$, i.e., there is no marketing strategy and pricing is the only decision. In addition, seller i dominates seller j if $b_i \le b_j$ and $a_i - b_i c_i - \ln b_i \ge a_j - b_j c_j - \ln b_j$, or equivalently, $e^{a_i - b_i c_i}/b_i \ge e^{a_j - b_j c_j}/b_j$. Finally, since we label competitive sellers in ascending order of their price sensitivity parameters, we can show that the value $a_i - b_i c_i - \ln b_i$ of competitive sellers are in descending order, i.e., $a_1 - b_1 c_1 - \ln b_1 > \ldots > a_{|\mathcal{C}_2|} - b_{|\mathcal{C}_2|} c_{|\mathcal{C}_2|} - \ln b_{|\mathcal{C}_2|}$ (see Lemma A.11 in the appendix).

Furthermore, we can show that for any pair of competitive sellers $i, j \in \mathcal{C}_2$ such that $i < j$, there exists a threshold $M_{i,j}$ such that seller i is a better candidate for the optimal seller i^* if $M < M_{i,j}$ (in the sense that $\pi_i(M) \le \pi_j(M)$), and seller j is a better candidate if $M > M_{i,j}$ (see Lemma A.12 in the appendix). Even though both seller i and j are non-dominated, one seller cannot be strictly better than the other for all budget values $M > 0$. Thus, even if a seller is not dominated by any other sellers, it is still possible that this seller may not be optimal for any $M > 0$. The construction of the values $M_{i,j}$ allows us to define a notion of "joint dominance".

Definition 4 (Joint dominance relation and candidate sellers). We call seller $j \in \mathcal{C}_2$ is jointly dominated by sellers $i, k \in \mathcal{C}_2$ if $i < j < k$ and $M_{i,j} < M_{j,k}$. Define the set of *candidate sellers* as

$$\mathcal{D}_2 \doteq \{i \in \mathcal{C}_2 : \text{seller } i \text{ is not jointly dominated by any pair of competitive sellers}\}.$$

Similar to before, we relabel the sellers so that the candidate sellers are in ascending order of their price sensitivity parameters, i.e., $b_1 < \ldots < b_{\mathcal{D}_2}$. Again, for any pair $i, j \in \mathcal{D}_2$, we have $b_i \neq b_j$.

Finally, we characterize the optimal solution to Problem 3 as follows.

Theorem 2. *Suppose Assumption 1 holds. The following are true:*

(i) For almost every $M > 0$, there is a generically unique optimal solution $(\mathbf{p}^*(M), \mathbf{x}^*(M))$ to Problem 3, which is one of $(\mathbf{p}(\mathbf{x}^i(M)), \mathbf{x}^i(M))$ for all $i \in [N]$.

(ii) The pair $(\mathbf{p}(\mathbf{x}^i(M)), \mathbf{x}^i(M))$ is optimal for some $M > 0$ if and only if seller $i \in \mathcal{D}_2$.

(iii) The optimal marketing strategy $\mathbf{x}^*$ is concentrated on one seller in $[N]$, and the optimal prices vector $\mathbf{p}^*$ satisfies the adjusted equal-markup property, *i.e., $p_i^* - c_i - \frac{1}{b_i} = \pi(\mathbf{p}^*(M), \mathbf{x}^*(M))$ for all $i \in [N]$.*

(iv) Given candidate sellers $i = 1, \ldots, |\mathcal{D}_2|$, there exists a sequence of thresholds $\{\mu_k\}_{0 \le k \le |\mathcal{D}_2|}$ with $\infty = \mu_0 > \mu_1 > \ldots > \mu_{|\mathcal{D}_2|-1} > \mu_{|\mathcal{D}_2|} = 0$ such that $(\mathbf{p}^*(M), \mathbf{x}^*(M)) = (\mathbf{p}(\mathbf{x}^i(M)), \mathbf{x}^i(M))$, if $M \in (\mu_i, \mu_{i-1})$.

(v) *Suppose the price sensitivity parameters of all sellers are identical, i.e.,* $b_i \equiv b$*, for all* $i \in [N]$*. The optimal marketing strategy* $\mathbf{x}^*$ *is concentrated on the seller* $i \in [N]$ *with largest* $a_i - bc_i$ *in* $[N]$*, and the optimal prices vector* $\mathbf{p}^*$ *satisfies the* equal-markup property*, i.e.,* $p_i^* - c_i = \pi(\mathbf{p}^*(M), \mathbf{x}^*(M))$ *for all* $i \in [N]$*.*

The adjusted equal markup property derives from a similar analysis, and yields similar insights, to the classical equal markup property under the MNL choice model, as studied in Li and Huh (2011).

Therefore, for every seller $i \in \mathcal{D}_2$, there exists some budget value M such that $(\mathbf{p}(\mathbf{x}^i(M)), \mathbf{x}^i(M))$ is optimal. As the available budget M increases, marketing effort allocations shifts among candidate sellers from the seller with the largest $e^{a_i - b_i c_i}/b_i$ to the seller with the smallest price sensitivity parameter b_i. On the contrary, sellers not in $\mathcal{D}_2$ can never be optimal, i.e., can never receive any marketing effort regardless of the amount of available budget M. Further implications of this result are discussed in the next subsection.

3.4. Summary of findings

In this section, we have examined a scenario where a platform makes centralized pricing and marketing decisions to maximize the total welfare (i.e., total profits) of all its sellers. The objective of maximizing total welfare can be thought of as an expression of utilitarianism, which tends to prioritize the overall welfare over that of individuals. The findings of this section indicate that such utilitarian pricing and marketing strategies can result in discrimination against underrepresented sellers, as illustrated concretely in Example 2.

The total profit-maximizing pricing strategy discriminates against sellers with larger price sensitivity parameters. Specifically, by the adjusted equal markup property (i.e., statement (iii) in Theorem 2), i.e., $p_i^* - c_i = 1/b_i + \pi^*$, sellers with larger price sensitivity parameters have a lower marginal profit. Zhang et al. (2021) have empirically analyzed the effect of the pricing algorithm of Airbnb on White hosts and Black hosts. They realize that Black hosts and White hosts face different demand curves for equivalent properties, and the demand for Black hosts' properties is more responsive to price changes than the demand for White hosts' properties. That is, underrepresented sellers usually have larger price-sensitivity parameters. Combining these facts together, our result shows that underrepresented sellers suffer from a smaller marginal profit even with equivalent products because they often have larger price sensitivity parameters due to customer discrimination.

The discriminatory behavior of the optimal marketing strategy has one extra layer of complexity beyond price sensitivity. The utilitarian marketing strategy discriminates sellers by defining a dominance relation, as shown in Definition 3. It does so in multiple ways. First, it discriminates against sellers with larger price sensitivity parameters because, as shown in Definition 3, sellers with larger price sensitivity parameters cannot dominate sellers with smaller price sensitivity parameters. As a result, sellers being discriminated against by customers are more likely to be dominated by other sellers. Furthermore, even if we assume customers are equally responsive to the price changes for all sellers, it still discriminates against sellers with lower attraction values e^{u_i}. This puts underrepresented sellers at a disadvantage due to customer discrimination. In Example 2, consider the counterfactual scenario that all iPod sellers have identical price-sensitivity parameters. Assume they have the same marginal costs as they are selling the same products. If the platform recommends prices to each seller with the goal of maximizing total profit, then, by Theorem 2, the resulting prices would not be discriminatory as the prices would be the same for all sellers. However, due to customer discrimination, underrepresented sellers may be perceived as having lower attraction values (as explained in Example 2), and this phenomenon triggers the platform to direct all marketing effort towards one light-skinned seller according to Theorem 2. This can exacerbate racial disparities.

4. Fair market share constraint with inequality-neutral platform

As discussed in the introduction, a significant amount of evidence suggests that underrepresented sellers frequently experience unjust market outcomes, such as unequal market shares, despite having competitive quality (Cox 2017, Edelman and Luca 2014, Bar and Zussman 2017). What would happen if the platform promoted underrepresented sellers in response to pressure from society to generate more equal market shares? In this section, we analyze the impact of this type of social norm by considering a fair market share constraint.

4.1. Model setup

We classify each seller into one of two groups based on certain sensitive attributes: a protected group $P \subset [N]$ and a non-protected group. The protected group is the group of sellers that society believes should benefit from being protected from potential customer discrimination. All other sellers are in the non-protected group.

To capture the scenario that unfair market outcomes prevail for protected sellers when a fair market share constraint is *not* imposed and the platform *does not* engage in a marketing strategy (what we call the *natural state*), we assume that the total market share of the protected group is lower than a societally-accepted proportion β in this natural state. Formally, we express this assumption as follows.

Assumption 2 **(Unfair market outcomes in the natural state)**. *For a given* $\beta < 1$,

$$\sum_{i\in P} \frac{e^{u_i}}{1+\sum_{j\in[N]} e^{u_j}} < \beta \sum_{i\in[N]} \frac{e^{u_i}}{1+\sum_{j\in[N]} e^{u_j}}, \tag{5}$$

or equivalently, $\sum_{i\in P} q_i < \beta \sum_{i\in[N]} q_i$.

Under the societal pressure to achieve a more equitable distribution of market share, suppose the platform strives to select a marketing strategy $\mathbf{x}$ that ensures that the total market share of the protected sellers is this β proportion of the total market share, i.e.,

$$\sum_{i\in P}\frac{e^{u_i+x_i}}{1+\sum_{j\in[N]}e^{u_j+x_j}}\geq\beta\sum_{i\in[N]}\frac{e^{u_i+x_i}}{1+\sum_{j\in[N]}e^{u_j+x_j}},\tag{6}$$

or equivalently, $\sum_{i\in P}s_i(\mathbf{x})\geq\beta\sum_{i\in[N]}s_i(\mathbf{x})$.

We call this the fair market share constraint. By Assumption 2, the fair market share constraint is violated if no marketing effort is made. If the platform's marketing budget is not large enough, it will not be able to comply with the fair market share constraint. Therefore, we assume the platform has a sufficient budget to honor the fair market share constraint. To avoid some degenerate cases, we assume there exists a strictly feasible solution.

Assumption 3 **(Enough marketing effort budget assumption)**. *Given fairness level β, the marketing effort budget M is large enough such that there exists a feasible marketing strategy to Problem 4 that strictly satisfies the fair market share constraint* (6)*.*

4.2. Analysis of the optimal marketing strategy

We consider the problem that the platform allocates limited marketing effort among all sellers in $[N]$ to maximize the total profits subject to the fair market share constraint. Recall that $y_i(\mathbf{x})\doteq e^{x_i}-1$ represents the multiplicative factor.

Problem 4 (Fairness-constrained total profit maximization problem). Suppose Assumptions 2 and 3 hold. Given a protected group $P\subset[N]$ and a level of fairness β, choose marketing strategy $\mathbf{x}$ to solve

$$\begin{aligned}\max_{\mathbf{x}\in\mathbb{R}^N}\quad&\sum_{i\in[N]}r_i\frac{e^{u_i+x_i}}{1+\sum_{j\in[N]}e^{u_j+x_j}}\\ \text{s.t.}\quad&\sum_{i\in P}\frac{e^{u_i+x_i}}{1+\sum_{j\in[N]}e^{u_j+x_j}}\geq\beta\sum_{i\in[N]}\frac{e^{u_i+x_i}}{1+\sum_{j\in[N]}e^{u_j+x_j}},\\ &\sum_{i\in[N]}y_i(\mathbf{x})\leq M,\\ &x_i\geq 0,\quad\text{for all } i=1,\dots,N.\end{aligned}$$

We first note that due to our Assumption 2 and fairness constraint (6), any feasible solution to Problem 4 must allocate at least some marketing effort to a seller in the protected group. As we did in the previous section with Problem 1, we transform Problem 4 into a linear program by considering the inverse marketing effort function (3).

Problem 5 (Linear fairness-regulated revenue-maximization problem). Given a protected group $P \subset [N]$ and a level of fairness β, choose market share vector $\mathbf{q}$ to solve

$$
\begin{aligned}
\max_{\mathbf{q}\in\mathbb{R}^N} \quad & \sum_{i\in[N]} r_i q_i \\
\text{s.t.} \quad & \sum_{i\in P} q_i \geq \beta \sum_{i\in[N]} q_i, \\
& \sum_{i\in[N]} e^{-u_i} q_i \leq (1-\sum_{i\in[N]} q_i)(M+N), \\
& q_i \geq e^{u_i}(1-\sum_{i\in[N]} q_i), \quad \forall i\in[N], \\
& \sum_{i\in[N]} q_i \leq 1, \\
& q_i \geq 0, \quad \forall i\in[N].
\end{aligned} \tag{7}
$$

The first constraint is the fair market share constraint. The second constraint is the budget constraint. The third constraint requires a non-negative marketing strategy $\mathbf{x}$. We show the following structure for the optimal marketing strategy $\mathbf{x}^*$.

Theorem 3. *Suppose Assumption 2 holds. The generically unique optimal solution $\mathbf{x}^*$ to Problem 4 satisfies one of the following cases:*

(i) marketing effort is concentrated on one seller, and this seller is protected;

(ii) one protected seller and one non-protected seller split all marketing effort;

(iii) two protected sellers split all marketing effort.

Moreover, if marketing effort is large enough, case (iii) does not occur.

We give examples to illustrate each case.

Example 3 (Case (i)). Consider the case that $r_i \equiv 1$ for all $i \in [N]$. Suppose i^* is the unique seller in $\arg\max_{i\in[N]} u_i$ and i^* is in the protected group P. Let $\mathbf{x}^*$ concentrate on seller i^*. We claim $\mathbf{x}^*$ is optimal to Problem 4. To see this, firstly notice that $\mathbf{x}^*$ is feasible to both Problem 1 and Problem 4. By Corollary 1, $\mathbf{x}^*$ is also optimal to Problem 1. Since the feasible region of Problem 4 is a subset of the feasible region of Problem 1, $\mathbf{x}^*$ is also optimal to Problem 4.

This is the case when one protected seller i^* is very popular, although other protected sellers have much lower attraction value due to Assumption 2.

Example 4 (Case (ii)). Consider the case that $r_i \equiv 1$ for all $i \in [N]$. Let i_P° and i_R° be the seller with the largest attraction value e^{u_i} in the protected and non-protected groups, respectively. Suppose $u_{i_P^\circ} < u_{i_R^\circ}$. Then, i_P° and i_R° split all of the available marketing effort under the optimal marketing strategy. (For the formal proof, see case (ii) of Theorem 4.) In this case, the platform's interest is solely promoting seller i_R° in the non-protected group. However, this is not feasible due to Assumption 2. Thus, the platform must spare some marketing effort for protected sellers to satisfy the fair market share constraint. The cheapest way to comply with the constraint is to solely promote one protected seller i_P°.

EXAMPLE 5 (CASE (III)). Consider the case that we have two protected sellers i_1 and i_2 and one non-protected seller i_r. Suppose that $r_{i_1} \approx 0$, u_{i_1} is large, r_{i_2} is large, $u_{i_2} \approx 0$, $r_{i_r} \approx 0$, and $u_{i_r} < u_{i_1}$. Suppose the total marketing budget M is not sufficient to satisfy the fair market share constraint if concentrated on seller i_2. It is not profitable to concentrate all marketing effort on seller i_1. Then, the optimal marketing strategy is to allocate all of the marketing effort between i_1 and i_2. The platform promotes i_2 to generate profits and promotes i_1 to honor the fair market share constraint. This example illustrates the intrinsic trade-off of the platform between profitability, i.e., larger marginal profit r_i, and popularity, i.e., larger attraction value e^{u_i}.

This result shows that even if the platform successfully implements a fair market share constraint in response to the societal pressure for fair market share distribution, only a fixed number of protected sellers (at most two) can benefit from the marketing strategy. In other words, the number of protected sellers that are allocated marketing effort remains constant and does not depend on the population of protected sellers or the marketing budget. Additionally, although the fair market share constraint can guarantee a reasonable proportion (determined by β) of the protected group's total market share, the proportion increase is mainly driven by the market shares of the one or two selected protected sellers. As a consequence, all other protected sellers experience a decrease in their market shares (see (4)). This marketing strategy is focused solely on addressing the inequality between the protected and non-protected groups. It ignores and, in fact, worsens the inequality within the protected group.

This practice may be viewed as an instance of "tokenism". Tokenism is the practice of making only a symbolic effort to be inclusive to members of underrepresented groups (Kanter 1977, Benton 2021, Snellman and Solal 2023). For example, Chang et al. (2019) examine how social norms and scrutiny affect decisions about adding members of underrepresented populations to groups. When groups are scrutinized, decision-makers try to match the diversity observed in peer groups due to impression management concerns, thereby conforming to the social norm. Their analysis of S&P 1500 boards reveals that significantly more boards include exactly two women (the social norm) than would be expected by chance. Chang et al. (2019) call this phenomenon "twokenism". They show that companies are less likely to add additional women to their boards once their boards have met the social norm for gender diversity by including two women. This phenomenon is more pronounced among more visible companies.

The analysis in this section demonstrates that when a platform promotes underrepresented sellers in response to pressure from fair market share constraints (a social norm), it can achieve compliance with the norm in a cost-effective way by promoting a maximum of two underrepresented sellers.

The promotion of a fixed number of underrepresented sellers is merely a "token" gesture designed to comply with the fairness constraint. Such a strategy can be seen as an instance of "twokenism."

4.3. Special case: Equal marginal profits

In the general setting, there is a possibility of "twokenism": two protected sellers split the marketing budget. In this subsection, we examine a special case where exactly one protected seller is promoted by the platform. In this case, we derive a further insight into the "discontinuous" nature of this promotional support that may see wild swings in which protected seller reaps benefits from the platform.

Our analysis proceeds in the special case that $r_j \equiv r > 0$ for all $i \in [N]$, which transforms the marketing effort allocation problem into a market share maximization problem. The motivation for this setting was discussed in the paragraph before Corollary 1 above.

Theorem 4. *Suppose Assumptions 2 and 3 hold and suppose further that $r_j \equiv r > 0$ for all $i \in [N]$. Let i_P° and i_R° be the seller with the largest basic utility u_i in the protected and non-protected groups, respectively. Then,*

(i) if $u_{i_P^\circ} > u_{i_R^\circ}$, the market-share maximization marketing strategy is to concentrate all marketing effort on i_P°;

(ii) if $u_{i_P^\circ} < u_{i_R^\circ}$, the market-share maximization marketing strategy is to split all marketing effort between i_P° and i_R°. In this case,

$$e^{x_{i_P^\circ}} - 1 = \frac{\beta(\sum_{j\in[N]} e^{u_j} + M e^{u_{i_R^\circ}}) - \sum_{j\in P} e^{u_j}}{(1-\beta)e^{u_{i_P^\circ}} + \beta e^{u_{i_R^\circ}}},$$

and $e^{x_{i_R^\circ}} - 1 = M - (e^{x_{i_P^\circ}} - 1)$.

Note that case (iii) in Theorem 3 does not arise in this context. This is due to the fact that when all sellers have identical marginal profits, the platform no longer faces a trade-off between profitability and popularity. Consequently, the platform only needs to focus on popularity, and there is no incentive to divide its marketing effort among two protected sellers. Theorem 4 further implies the following non-monotone and discontinuous behavior of optimal marketing strategy.

Corollary 2 **(Discontinuity of marketing strategy)**. *Suppose Assumptions 2 and 3 hold and suppose further that $r_j \equiv r > 0$ for all $i \in [N]$. Let i_P° and i_R° be the sellers with the largest basic utility u_i in the protected group and non-protected group, respectively. Then, given the optimal marketing strategy $\mathbf{x}^*$, the amount of marketing effort $x^*_{i_P^\circ}(u_{i_P^\circ})$ of seller i_P° exhibits the following non-monotone and discontinuous behavior as a function of $u_{i_P^\circ}$:*

(i) $x^*_{i_P^\circ}(u_{i_P^\circ}) < \ln(M+1)$ and is decreasing in $u_{i_P^\circ}$ if $u_{i_P^\circ} < u_{i_R^\circ}$; and

(ii) $x^*_{i_P^\circ}(u_{i_P^\circ}) = \ln(M+1)$ if $u_{i_P^\circ} > u_{i_R^\circ}$,

where $\ln(M+1)$ is the largest possible amount of marketing effort given budget M.

From Theorem 4, we know that seller i_P° and i_R° split all marketing effort. Corollary 2 answers the question of how the amount of effort allocated to i_P° changes as the seller i_P° improves the basic utility $u_{i_P^\circ}$. Firstly, as shown in statement (i), when the basic utility $u_{i_P^\circ}$ of the protected seller i_P° is not the highest among all sellers, the protected seller i_P° will receive less effort as seller i_P° improves its basic utility $u_{i_P^\circ}$. That is, the effort allocated to seller i_P° is decreasing in the basic utility. This happens because the platform does not need to help this seller as much; their improving basic utility is attracting sufficient customers.

However, the change in the marketing effort allocation is not continuous, as shown in statement (ii). As seller i_P° becomes the seller with the highest basic utility among all sellers, seller i_P° will suddenly be allocated all of the marketing effort. When the basic utility $u_{i_P^\circ}$ of protected seller i_P° is higher than the basic utility u_i of all the other sellers $i \neq i_P^\circ$, then the protected seller i_P° receives all of the marketing effort.

One may find this sudden "jump" in marketing effort allocation to be unnatural at first glance. The discontinuity is a result of a concentrated marketing strategy. Under a concentrated marketing strategy, whoever has the highest basic utility receives all of the marketing effort. Therefore, before the protected seller i_P° becomes the seller with the highest basic utility, improving the basic utility can only make it easier for the concentrated marketing strategy to honor the fair market share constraint, which explains the decreasing behavior of $x^*_{i_P^\circ}(u_{i_P^\circ})$; and when the protected seller i_P° becomes the seller with the highest basic utility, this seller gets all of the marketing effort.

This "winner takes all" among members of the protected group leads to additional complications. For example, suppose that a protected seller is currently receiving a promotion from the platform, but suppose the platform updates its policies as the basic utilities $\mathbf{u}$ and marginal profits $\mathbf{r}$ change. Then, there is an incentive for protected sellers to compete with one another to become attractive enough to receive the concentrated marketing effort of the platform. This internal competition among those in a protected group can lead to disunity and mistrust among its members, further weakening their collective power. Although not modeled explicitly in our analysis, this additional effect of tokenistic allocation schemes should be considered a societal cost and may contribute further to the marginalization of certain groups.

5. Inequality-averse regime

In this section, we assume that the platform is "inequality averse", which is reflected in its utility structure. In Section 5.1, we demonstrate that various widely used fairness concepts can be consolidated under the general notion of α-fairness, where α controls the degree of aversion to inequality.

Then, in Section 5.2, we analyze the platform's problem with various degrees of inequality aversion and contrast its outcomes with earlier findings from the previous two sections.[6]

5.1. Notions of fairness

Thus far, we have shown that a profit-maximization marketing strategy tends to be concentrated. Even if the platform promotes protected sellers in response to pressure from society that requires a minimum market share for the protected group, the resulting marketing strategy only promotes a limited number of protected sellers. To achieve more "even" market outcomes, a stronger notion of fairness appears to be necessary. In the following subsection, we discuss several notions of fairness.

The utilitarianism criterion is the most commonly used benchmark for fairness, whereby the platform seeks to maximize the total utility of all sellers, which is the total profit $\sum_{i\in[N]}\pi_i(\mathbf{x})$ in our setting. Since the sum of utilities is neutral to the potential inequalities in the utility distribution among sellers, the corresponding optimal strategy may prioritize the total utility at the expense of the interests of certain individuals. As we have seen, maximizing total profit leads to a concentrated marketing strategy.

A frequently applied notion of fairness is the Nash standard (Nash 1950). A transfer of resources between two agents is justified if the gainer's utility increases by a larger percentage than the loser's utility decreases. Bertsimas et al. (2011) extend this notion to settings with multiple agents. With multiple agents, a fair allocation of utilities $\mathbf{v}^* \doteq (v_1^*,\ldots,v_N^*)$ in the Nash sense should be such that, if compared to any other feasible allocation of utilities $\mathbf{v} \doteq (v_1,\ldots,v_N)$, the aggregate proportional change is less than or equal to 0. In mathematical terms, given the utility set V, the Nash fair allocation $\mathbf{v}^*$ satisfies

$$\sum_{i\in[N]} \frac{v_i - v_i^*}{v_i^*} \le 0, \quad \forall \mathbf{v}\in V.$$

Bertsimas et al. (2011) show that in case U is convex, the Nash fair allocation of utilities $\mathbf{u}^*$ is the unique optimal solution of the problem

$$\max_{\mathbf{v}\in V} \quad \sum_{i\in[N]} \ln v_i,$$

since the necessary and sufficient first-order optimality condition for this problem is exactly the Nash standard of comparison principle for N agents.

[6] We do not pursue the question of joint optimization of pricing and marketing effort allocation in this section. An analysis of this more general problem introduces a lot more technical complexity (the underlying problem is not convex and is even difficult to solve numerically), and we felt it did not yield sufficient insight to warrant addition.

The utilitarianism criterion and Nash standard are unified in the notion of α-fairness (Bertsimas et al. 2012). According to α-fairness, the platform decides the allocation of utilities $\mathbf{v}$ by maximizing the constant elasticity social welfare function W_α defined as

$$W_\alpha(\mathbf{v}) \doteq \begin{cases} \sum_{j\in[N]} \frac{v_j^{1-\alpha}}{1-\alpha}, & \alpha \in [0,1), \\ \sum_{j\in[N]} \ln v_j, & \alpha = 1. \end{cases}$$

Note that when $\alpha = 0$, this scheme corresponds to the utilitarianism solution, while when $\alpha = 1$, this scheme corresponds to the Nash solution. In our setting, the utility v_i of seller i is given by their profit π_i. Given the profit vector $\boldsymbol{\pi} \doteq (\pi_1, \ldots, \pi_N)$, for $\alpha < 1$, we can factorize the α-fairness as follows

$$\begin{aligned} W_\alpha(\boldsymbol{\pi}) &= \frac{1}{1-\alpha} \sum_{i\in[N]} \pi_i^{1-\alpha} \\ &= \frac{1}{1-\alpha} (\sum_{i\in[N]} \pi_i)^{1-\alpha} [\sum_{i\in[N]} (\frac{\pi_i}{\sum_{j\in[N]} \pi_j})^{1-\alpha}]. \end{aligned} \tag{8}$$

In equation (8), the value $\sum_{i\in[N]} \pi_i$ represents the total profits, and the value $\pi_i/\sum_{j\in[N]} \pi_j$ represents the profit share of seller i in the total profits. If we increase the profit π_i of each seller i by a constant factor, the second term $\sum_{i\in[N]}(\pi_i/\sum_{j\in[N]} \pi_j)^{1-\alpha}$ remains unchanged as we do not alter the profit distribution. However, the total profits increase due to the higher individual profits, which in turn results in a higher α-fairness value. On the other hand, given total profits $\sum_{i\in[N]} \pi_i$, the α-fairness is maximized if the total profits are distributed uniformly among sellers in the sense that $\pi_i/\sum_{j\in[N]} \pi_j = 1/N$ for all $i \in [N]$. In conclusion, α-fairness balances the preference for a larger total utility and the preference for a more equal distribution of utilities. The balance of two preferences is controlled by the parameter α. A larger α represents a stronger preference for equal distribution. It holds since for a larger α, the marginal increase in the profits of sellers with small profits results in a more significant increase in the α-fairness value. In the limit when $\alpha = 1$, the total profits $\sum_{i\in[N]} \pi_i$ do not influence the value of α-fairness. Therefore, the parameter α is usually called the inequality aversion parameter. For a more detailed discussion, see Bertsimas et al. (2012).

In the following subsections, we consider the case that the platform is inequality averse. Specifically, we study the problem of allocating a finite amount of total marketing effort to maximize α-fairness. We show that the optimal marketing strategy with a high enough level of inequality aversion can result in a more even allocation of marketing effort and more even distribution of market share.

5.2. Analysis of the inequality-averse marketing strategy

Consider the problem of the inequality-averse platform allocating limited marketing effort among all sellers in $[N]$ to maximize α-fairness. Without loss of generality, we allocate marketing effort among all sellers in $[N]$. Our results can be easily extended to the setting that we only allocated marketing effort among protected sellers $P \subset [N]$. Recall that $y_i(\mathbf{x}) \doteq e^{x_i} - 1$ represents the multiplicative factor.

Problem 6 (α-fair marketing effort allocation problem). Define the α-fair objective as

$$f^\alpha(\mathbf{x}) \doteq \begin{cases} \frac{1}{1-\alpha} \sum_{i \in [N]} (r_i \frac{e^{u_i + x_i}}{1 + \sum_{j \in [N]} e^{u_j + x_j}})^{1-\alpha}, & \forall \alpha \in [0,1), \\ \sum_{i \in [N]} \ln (r_i \frac{e^{u_i + x_i}}{1 + \sum_{j \in [N]} e^{u_j + x_j}}), & \alpha = 1. \end{cases}$$

Choose marketing strategy $\mathbf{x}$ to solve

$$\begin{aligned} \max_{\mathbf{x} \geq 0} \quad & f^\alpha(\mathbf{x}) \\ \text{s.t.} \quad & \sum_{i \in [N]} y_i(\mathbf{x}) \leq M. \end{aligned}$$

Again, using inverse marketing effort function (3), we transform Problem 6 into a convex program.

Problem 7 (Convex fair marketing effort allocation problem). Define the α-fair objective as[7]

$$f^\alpha(\mathbf{q}) \doteq \begin{cases} \frac{1}{1-\alpha} \sum_{i \in [N]} (r_i q_i)^{1-\alpha}, & \forall \alpha \in [0,1) \\ \sum_{i \in [N]} \ln (r_i q_i), & \alpha = 1. \end{cases}$$

Choose market share vector $\mathbf{q}$ to solve

$$\begin{aligned} \max_{\mathbf{q} \in \mathbb{R}^N} \quad & f^\alpha(\mathbf{q}) \\ \text{s.t.} \quad & \sum_{i \in [N]} e^{-u_i} q_i \leq (1 - \sum_{i \in [N]} q_i)(M + N), \\ & q_i \geq e^{u_i} (1 - \sum_{i \in [N]} q_i), \quad \forall i \in [N], \\ & \sum_{i \in [N]} q_i \leq 1, \\ & q_i \geq 0, \quad \forall i \in [N]. \end{aligned} \tag{9}$$

This is a convex optimization problem (since f^α is a concave function of $\mathbf{q}$ for all $\alpha \in [0,1]$ and all constraints are linear in $\mathbf{q}$). By formulating the problem as a convex optimization, we can utilize Karush-Kuhn-Tucker (KKT) conditions to obtain structural insights into the optimal market shares.

5.2.1. Special case: Analysis of the Nash standard ($\alpha = 1$) In Section 3, we analyzed the profit-maximization marketing strategy, which corresponds to the utilitarian criterion where the platform has the lowest level of inequality aversion, i.e., $\alpha = 0$. In the following subsection, we

[7] We abuse notation slightly by allowing both $\mathbf{x}$ and $\mathbf{q}$ to be arguments of f^α.

analyze the opposite extreme, where the platform has the highest level of inequality aversion, i.e., $\alpha = 1$. This case corresponds to the Nash standard.

We first show that with the Nash standard, the optimal marketing strategy is evenly distributed in the following sense.

PROPOSITION 3. *Sort the basic utilities in ascending orders, i.e.,* $u_1 \leq u_2 \leq \ldots \leq u_N$ *and suppose* $\alpha = 1$*, i.e., Nash standard holds. Then, the optimal marketing strategy* $\mathbf{x}^*$ *(i.e., optimal solution to Problem 6) satisfies the following properties:*

(i) sellers with lower basic utility get more marketing effort, i.e., $x_1^* \geq \ldots \geq x_N^*$*;*

(ii) for any marketing budgets M*, there exists* $k \in [N]$ *(which depends on* M*) such that the set of sellers with positive marketing effort is given by* $\{1, \ldots, k\}$*.*

(iii) as the marketing budget M *increases, the largest index* k *of a seller that is allocated a positive amount of marketing effort increases from* 1 *to* N*. Specifically, for* M *sufficiently large, all sellers are allocated positive marketing effort, i.e.,* $x_i^* > 0$ *for all* $i \in [N]$*.*

(iv) if the marketing marketing budget M *is large enough, the resulting market share* $\mathbf{q}^*(\mathbf{x}^*)$ *and profit* $\pi^*(\mathbf{x}^*)$ *satisfies, for any pair of* $i, j \in [N]$*,*

$$\frac{q_i^*(\mathbf{x}^*)}{q_j^*(\mathbf{x}^*)} = \frac{e^{-u_j} + M + N}{e^{-u_i} + M + N} \to 1 \quad \textit{and} \quad \frac{\pi_i^*(\mathbf{x}^*)}{\pi_j^*(\mathbf{x}^*)} = \frac{r_i(e^{-u_j} + M + N)}{r_j(e^{-u_i} + M + N)} \to \frac{r_i}{r_j} \quad \textit{as } M \to \infty.$$

We compare the optimal marketing strategy under the utilitarian criterion (i.e., $\alpha = 0$) and Nash Standard (i.e., $\alpha = 1$) as follows. Recall that under the utilitarian criterion, the optimal marketing strategy concentrates all of its marketing effort on one seller. In particular, when the total budget of M is small enough, the optimal marketing strategy concentrates on the seller with the largest basic utility. As available budget M increases, the concentrated allocation of marketing effort shifts towards sellers with higher marginal profits in the order characterized by Theorem 1. Dominated sellers never receive a positive allocation of marketing effort. By contrast, under the Nash standard, sellers with smaller basic utility always get more marketing effort, regardless of the amount of available budget or the marginal profit. Moreover, under the Nash standard, the number k of sellers who can get promoted increases as the available budget M increases. When M is large enough, every seller gets promoted, and the seller with a smaller basic utility gets more marketing effort. In the utilitarian case, the number of sellers who are allocated positive marketing effort never changes.

As the available budget M increases, the ratio of market shares between two arbitrary sellers $i, j \in [N]$ converges to 1 (from part (iv) of the theorem). Hence, by adjusting the total marketing budget, the platform can control the level of inclusivity in the market share distribution. The following example shows that the Nash standard can avoid the potential harm caused by the concentrated marketing strategy described in Example 2.

EXAMPLE 6 (IPOD SELLING ON ONLINE MARKET (DOLEAC AND STEIN 2013)). In the same setting described in Example 2, by Proposition 3, under the Nash standard, the dark-skinned sellers get more marketing effort than the light-skinned sellers. Moreover, by Proposition 3(iv), with a large enough available budget M, the market shares and attraction values of dark-skinned sellers and light-skinned sellers converge to the same value.

5.2.2. Analysis of the general case ($\alpha \in (0,1)$) For the intermediate levels of inequality aversion where $\alpha \in (0,1)$, we show that if α is large enough, the optimal marketing strategy exhibits the same properties as the optimal marketing strategy under Nash standard described in Proposition 3.

THEOREM 5 **(Inclusive marketing strategy)**. *Sort the attraction values in ascending orders, i.e., $u_1 \le u_2 \le \ldots \le u_N$. For $\alpha \in (0,1)$ large enough, the optimal marketing strategy $\mathbf{x}^*$ (i.e., optimal solution to Problem 6) satisfies the following properties:*

(i) sellers with lower attraction value gets more marketing effort, i.e., $x_1^* \ge \ldots \ge x_N^*$;

(ii) the set of sellers with a positive allocation of marketing effort is given by $\{1,\ldots,k\}$ for some $k \in [N]$, i.e., $\{i \in [N] : x_i^* > 0\} = \{1,\ldots,k\}$ for some $k \in [N]$;

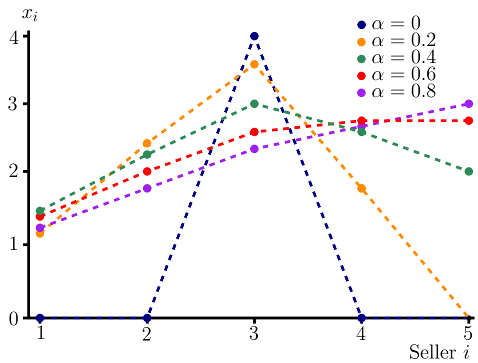
(iii) as the marketing budget M increases, the largest index k of a seller that is allocated a positive amount of marketing effort increases from 1 to N. Specifically, if M is large enough, all sellers get positive marketing effort, i.e., $x_i^* > 0$ for all $i \in [N]$.

(iv) if the marketing budget M is large enough, the resulting market share $\mathbf{q}^*(\mathbf{x}^*)$ and profit $\pi^*(\mathbf{x}^*)$ satisfies, for any pair $i,j \in [N]$,

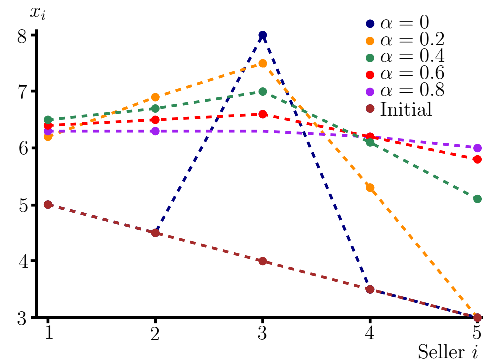
$$\frac{q_i^*(\mathbf{x}^*)}{q_j^*(\mathbf{x}^*)} = \left(\frac{r_i}{r_j}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{e^{-u_j}+M+N}{e^{-u_i}+M+N}\right)^{\frac{1}{\alpha}} \quad \text{and} \quad \frac{\pi_i^*(\mathbf{x}^*)}{\pi_j^*(\mathbf{x}^*)} = \frac{r_i^{\frac{1}{\alpha}}(e^{-u_j}+M+N)^{\frac{1}{\alpha}}}{r_j^{\frac{1}{\alpha}}(e^{-u_i}+M+N)^{\frac{1}{\alpha}}}.$$

This result illustrates that with a reasonable level of inequality aversion α, the corresponding optimal marketing strategy promotes the sellers with smaller basic utilities first. As the available marketing budget increases, the number k of promoted sellers increases from 1 to N. Eventually, all sellers get promoted, and the sellers with smaller basic utility receive more marketing effort. In contrast with the concentrated marketing strategy described in Theorem 1 and the tokenist strategy in Theorem 3, we can fairly call this marketing strategy "inclusive".

Moreover, the inequality in the resulting market outcomes (i.e., market share distribution and profit distribution) is determined by the platform's level of inequality aversion. When the level of inequality aversion is high enough, the marketing strategy results in a more even market outcome. For example, as M increases to ∞, the ratio of market shares between two arbitrary sellers $i,j \in [N]$ converges to $(r_i/r_j)^{1-1/\alpha}$. This limit converges to 1 as α increases to 1. On the contrary, when the level of inequality aversion is low (i.e., α is close to 0), we show in the Section 6 that the corresponding marketing strategy can exacerbate the disparity in market outcomes.



(a) Effort allocations



(b) Seller utilities

Figure 2 Analysis of optimal marketing strategies

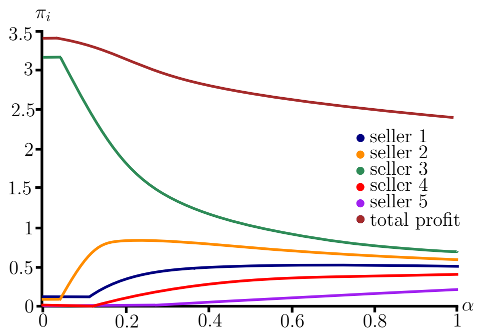
6. Numerical experiments

We run a simulation to test the implication of different inequality aversion parameter α. To make the figures we present more readable, we only consider five sellers. We randomly generated their attraction values u_i and marginal profit r_i and sorted them in descending order of their basic utilities u_i, i.e., $u_1 \geq \ldots \geq u_5$. For a description of the instance we represent in our figures, see Section EC.4. This instance was representative of many other instances we explored. Note that sellers 4 and 5 have low basic utilities either because they are under discrimination or they offer a low-quality offering. We solve Problem 7 with the package `cvxpy` in python, which is appropriate since Problem 7 is a convex optimization problem.

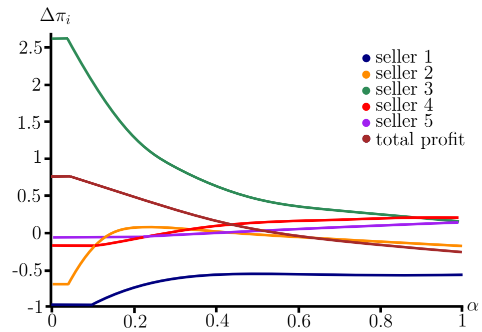
First, we analyze the impact of the inequality aversion parameter α on the optimal marketing strategy. Findings are illustrated in Figure 2. Figure 2a shows the optimal marketing strategy among sellers across five different levels of inequality aversion. From Figure 2a, when $\alpha = 0$, i.e., this is the total profit maximization problem, all marketing effort concentrates on seller 3, which is consistent with the Proposition 1. As the level of inequality aversion increases from 0 to 0.8, sellers with lower basic utility u_i get more and more marketing effort (e.g., seller 4 and seller 5). Specifically, when the level of inequality aversion is high enough (e.g., $\alpha = 0.6$ or 0.8), the marketing effort is in the reverse order of the basic utility (as shown in statement (i) in Theorem 5).

Figure 2b shows sellers' total utility given the optimal marketing strategy, i.e., $u_i + x_i$. The brown line represents the basic utility u_i in descending order. Notice that when the level of inequality aversion is low (e.g., $\alpha = 0$ or $\alpha = 0.2$), the total utilities of sellers are highly differentiated. There is a high peak at seller 3, and the total utilities of seller 4 and 5 are low. However, as the level of inequality aversion increases from 0 to 0.8, the curve of total utilities gradually becomes flat. When the level of inequality aversion is high enough (e.g., $\alpha = 0.8$), the curve becomes almost horizontal, i.e., all of the sellers have nearly equal total utilities $u_i + x_i$.

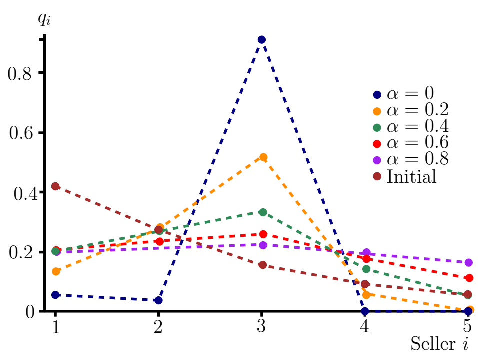
To conclude, with a large enough level of inequality aversion, every seller gets promoted, and sellers with low basic utility get more marketing effort (as shown in statement (i) and (iii) Theorem 5). This avoids the tokenism-type issue found in Section 4. However, on the negative side, the marketing strategy with an overly high level of inequality aversion possibly makes the sellers with low quality as attractive as the sellers with high quality. This fundamentally distorts the nature of the marketplace.



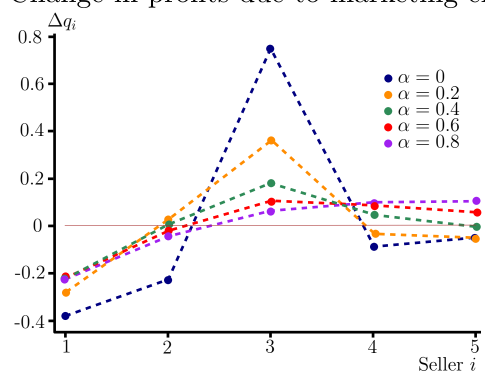
(a) Profits under optimal marketing effort



(b) Change in profits due to marketing effort



(c) Market shares under optimal marketing effort allocations



(d) Change in market shares due to marketing effort

Figure 3 **Further analysis of optimal market strategies**

Next, we analyze the impact of the inequality aversion parameter α on the whole market in terms of profits and market shares. Findings are illustrated in Figure 3. Figure 3a shows the profits of sellers under the optimal marketing strategy with different levels of inequality aversion. We find the following patterns:

(i) The total profit is decreasing as α increases from 0 to 1, as indicated by the brown line.

(ii) When the level of inequality aversion is low, profit distribution is concentrated in the sense that seller 3 gets almost all of the profits, as indicated by the green line.

(iii) As the level of inequality aversion increases, the profits of all sellers converge to a similar level (as shown in statement (iv) of Theorem 5).

(iv) As the level of inequality aversion increases, profits of sellers with low basic utilities are increasing.

Similar findings are found in market share distribution as shown in Figure 3c. When the level of inequality aversion is low, the market share distribution is even more concentrated than the initial state without marketing strategy. Therefore, marketing strategy with low α could exacerbate the disparity in market shares. As α increases, market shares of all sellers converge to a similar level (see statement (iv) in Theorem 5).

Figure 3b shows the change of profits due to marketing strategy, i.e., the profit with marketing effort minus the initial profit without marketing effort. The brown line depicts the change in total profit. As the level of inequality aversion becomes too high (i.e., at the point around $\alpha = 0.6$), the change of total profit becomes negative, which means the marketing strategy hurts the total profit. This suggests a conflict between efficiency and fairness in the market. When the α is small, the changes in profits for sellers with low basic utilities (i.e., seller 4 and 5) are negative, which means that a marketing strategy with low level of inequality aversion hurts the profits of sellers at a disadvantage. When α is large enough, the changes in profits for sellers with low basic utilities become positive. This implies that sellers at a disadvantage benefit from the marketing strategy with a high level of inequality aversion in terms of profits. Similar findings are found in the market shares as shown in Figure 3d.

From our previous analysis, we conclude that:

(i) marketing strategies with a low level of inequality aversion exacerbate the market disparities in terms of total utilities, market shares, and profits;

(ii) marketing strategies with a high level of inequality aversion result in almost total even market outcomes in terms of total utilities, market shares, and profits. In this case, the marketing strategy possibly promotes sellers with low quality.

7. Conclusion

In this paper, we analyzed an MNL model with marketing effort as a decision variable to study the nature of marketing effort allocation under fairness concerns. Our results show that utilitarian maximization yields concentrated marketing strategies that become "tokenistic" under fairness constraints. We see inclusive marketing efforts only when marketers have a high level of inequality aversion.

There remain many future research directions to explore. A natural extension would be a scenario where there are competing platforms where one (or both) are concerned with the impact of customer discrimination. An intriguing scenario might be for platforms to "compete" for sellers that add diversity to their platforms. A natural technical extension would be to analyze the setting where the pricing decision is decentralized to suppliers, which is common on some platforms. This consideration adds much technical complexity, as it results in a Stackelberg-Nash game that is delicate to analyze. Finally, we have examined a scenario with a homogenous customer base (outside of the heterogeneity coming from the random utility model behind the MNL framework). A more general model might consider multiple customer populations that are differentiated by which groups of sellers they are biased for or against. Analyzing this scenario could provide further insight into the impact of how one cultural group of customers might favor transacting with sellers of the same cultural group and how demands for "fairness" might disrupt such practices and incentivize a mixing of cultural interactions.

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Online appendix for "Allocating marketing effort under customer discrimination"

EC.1. Proofs of Section 3

EC.1.1. Proof of Proposition 1

Since Problem 2 is a linear program, it admits a generically unique optimal basic feasible solution $\mathbf{q}^* \in \mathbb{R}^N$. That is, there should be at least N constraints that are tight at $\mathbf{q}^*$.

Suppose $\sum_{i\in[N]} q_i^* = 1$. Then, the budget constraint becomes $\sum_{i\in[N]} e^{-u_i} q_i^* \le 0$, which further implies $q_i^* = 0$ for all $i \in [N]$. This contradicts the assumption that $\sum_{i\in[N]} q_i^* = 1$. Thus, $\sum_{i\in[N]} q_i^* < 1$.

Suppose $q_i^* = 0$ for some $i \in [N]$. Then, the non-negative marketing effort constraint of seller i becomes $0 \ge e^{u_i}(1 - \sum_{j\in[N]} q_j^*)$. This further implies $\sum_{j\in[N]} q_j^* = 1$, which is impossible. Thus, $q_i^* > 0$ for all $i \in [N]$.

Suppose the budget constraint is not tight, i.e., $\sum_{i\in[N]} e^{-u_i} q_i^* < (1 - \sum_{i\in[N]} q_i^*)(M+N)$. This is equivalent to $\sum_{i\in[N]}(e^{-u_i} + M + N) q_i^* < M + N$. Suppose we increase the value of q_i^* for some $i \in [N]$ by a small amount $\epsilon > 0$. Denote the new solution as $\mathbf{q}'$. If ϵ is small enough, $\mathbf{q}'$ is still feasible. To see this, since $\sum_{i\in[N]} e^{-u_i} q_i^* < (1 - \sum_{i\in[N]} q_i^*)(M+N)$ by assumption and we have already shown that $\sum_{j\in[N]} q_j^* < 1$, if ϵ is small, these two constraints are still satisfied by $\mathbf{q}'$. The non-negative marketing effort constraints and non-negativity constraints $q_i^* > 0$ are still feasible at $\mathbf{q}'$. Thus, the solution $\mathbf{q}'$ is still feasible. However, $\sum_{j\in[N]} r_j q_j' = r_i\epsilon + \sum_{j\in[N]} r_j q_j^* > \sum_{j\in[N]} r_j q_j^*$. This contradicts the fact that $\mathbf{q}^*$ is optimal. Thus, the budget constraint must be tight at $\mathbf{q}^*$, i.e., $\sum_{i\in[N]} e^{-u_i} q_i^* = (1 - \sum_{i\in[N]} q_i^*)(M+N)$.

Suppose all the non-negative marketing effort constraints are tight at $\mathbf{q}^*$, i.e., $q_i^* = e^{u_i}(1 - \sum_{j\in[N]} q_j^*)$ for all $i \in [N]$. This is equivalent to $e^{-u_i} q_i^* = 1 - \sum_{j\in[N]} q_j^*$, $\forall i \in [N]$. If we sum them together, we have $\sum_{i\in[N]} e^{-u_i} q_i^* = (1 - \sum_{j\in[N]} q_j^*) N < (1 - \sum_{i\in[N]}) q_i^* (M+N)$. Thus, the budget constraint is not tight, which is impossible. Therefore, there exists at least one non-negative marketing effort constraint that is not tight.

In conclusion, at the generically unique optimal basic feasible solution $\mathbf{q}^*$, there are total N constraints that are tight. One of the tight constraints is the budget constraint, and the remaining $N-1$ tight constraints are non-negative marketing effort constraints.

Let seller i^* be the only seller such that the corresponding non-negative marketing effort constraint is not tight. By the tight budget constraint $\sum_{i\in[N]} e^{-u_i} q_i^* = (1 - \sum_{i\in[N]} q_i^*)(M+N)$ and $N-1$ tight non-negative marketing effort constraints $e^{-u_i} q_i^* = 1 - \sum_{j\in[N]} q_j^*$ for all $i \ne i^*$, we have $e^{-u_{i^*}} q_{i^*}^* = (1 - \sum_{j\in[N]} q_j^*)(M+1)$. Therefore, we have $-u_{i^*} + \ln q_{i^*}^* - \ln(1 - \sum_{j\in[N]} q_j^*) = \ln(M+1)$

and $-u_i+\ln q_i^*-\ln(1-\sum_{j\in[N]}q_j^*)=0$ for all $i\neq i^*$. By the inverse marketing effort function (3), we have $x_{i^*}^*=\ln(M+1)$ and $x_i^*=0$ for all $i\neq i^*$. The total profit at $\mathbf{x}^*$, which is the optimal value of Problem 1, is given by

$$R_{i^*}=\frac{Mr_{i^*}e^{u_{i^*}}+\sum_{j\in[N]}r_je^{u_j}}{Me^{u_{i^*}}+1+\sum_{j\in[N]}e^{u_j}}.$$

EC.1.2. Proof of Theorem 1

To start with, we show that if the marginal profit r_j of seller j is smaller than the total profit π of all sellers without employing a marketing strategy, then it is never optimal to concentrate all marketing effort on seller j, i.e., $\mathbf{x}^j(M)$ is not optimal for any $M>0$.

Lemma A.1 **(Positions of marginal profits)**. *As M increases from 0 to ∞,*

(i) for each seller i such that $r_i<\pi$, $\pi(\mathbf{x}^i(M))$ decreases from π to r_i;

(ii) for each seller i such that $r_i>\pi$, $\pi(\mathbf{x}^i(M))$ increases from π to r_i.

Proof of Lemma A.1. If the platform concentrates all effort on seller i (i.e., the marketing strategy is $\mathbf{x}^i(M)$), then the net gain $G(\mathbf{x}^i(M))$ of total revenue given marketing strategy $\mathbf{x}^i(M)$ is given by

$$G(\mathbf{x}^i(M))\doteq\sum_{k\in[N]}r_k(q_k(\mathbf{x}^k(M)-s_k)=(r_i-\pi)\frac{Ms_i}{1+Ms_i}, \tag{A.1}$$

where the last equality is obtained by some algebraic operations. If $r_i<\pi$ (resp. $r_i>\pi$), $G(\mathbf{x}^i(M))$ is decreasing (resp. increasing) from 0 to $r_i-\pi$ as M increases from 0 to ∞, which implies statement (i) (resp. statement (ii)). □

Intuitively, it is always better to concentrate on seller i with marginal profit $r_i>\pi$. Thus, we have narrowed down the candidates for the optimal marketing strategy to $\{\mathbf{x}^i(M):r_i>\pi,i\in[N]\}$. Next lemma shows that if seller j is dominated by any other sellers according to Definition 1, then $\mathbf{x}^j(M)$ is not optimal for any $M>0$.

Lemma A.2 **(Consequence of dominance)**. *If seller i dominates seller j, then $\pi(\mathbf{x}^i(M))>\pi(\mathbf{x}^j(M))$ for any $M>0$. Specifically, if seller j is dominated by some seller i, $\mathbf{x}^j(M)$ cannot be optimal for any $M>0$.*

Proof of Lemma A.2. Suppose seller i dominates seller j, i.e., $r_i>r_j>\pi$ and $(r_i-\pi)s_i>(r_j-\pi)s_j$. If $s_i\geq s_j$, $G(\mathbf{x}^i(M))=(r_i-\pi)\frac{Ms_i}{1+Ms_i}>(r_j-\pi)\frac{Ms_j}{1+Ms_j}=G(\mathbf{x}^j(M))$ for any $M>0$. If $s_i<s_j$, $G(\mathbf{x}^i(M))>G(\mathbf{x}^j(M))$ is equivalent to

$$\frac{(r_i-\pi)s_i}{(r_j-\pi)s_j}>\frac{1+Ms_i}{1+Ms_j},$$

which holds for any $M>0$, since left hand side is larger than 1 while right hand side is smaller than 1 for any $M>0$. Hence, $\pi(\mathbf{x}^i(M))>\pi(\mathbf{x}^j(M))$ for any $M>0$. □

So far, we have identified two necessary conditions for $\mathbf{x}^i(M)$ to be optimal: (i) $r_i > \pi$; (ii) seller i is not dominated by any other sellers. Therefore, to find the optimal marketing strategy $\mathbf{x}^*(M)$, we only need to check the sellers with above-average marginal profit who are not dominated by any other sellers, i.e., sellers are in the competitive set in Definition 1. The competitive set $\mathcal{C}$ is never empty because, for any $M > 0$, the optimal seller i^* is always in it, since $\mathcal{C}$ is defined by two necessary conditions for $\mathbf{x}^*(M)$. Next lemma demonstrates the relation among competitive sellers.

Lemma A.3 **(Relation among competitive sellers)**. *Competitive sellers satisfy:*
(i) $(r_1 - \pi)s_1 < \ldots < (r_m - \pi)s_m$; *and*
(ii) $s_1 < \ldots < s_m$, *or equivalently,* $u_1 < \ldots < u_m$.

Proof of Lemma A.3. (i) For the sake of contradiction, suppose there exists a pair of sellers $i, j \in \mathcal{C}$ such that $i < j$ and $(r_i - \pi)s_i \geq (r_j - \pi)s_j$. Due to the labeling method of competitive set $\mathcal{C}$, $r_i > r_j$. Hence, seller i dominates seller j. This contradicts the fact that sellers i, j are competitive sellers.

(ii) For any pair of sellers $i, j \in \mathcal{C}$ such that $i < j$, due to the labelling and statement (i), $r_i > r_j$ and $(r_i - \pi)s_i < (r_j - \pi)s_j$. Hence, $s_i < s_j$, or equivalently, $u_i < u_j$. □

Since sellers in $\mathcal{C}$ do not dominate each other, the seller with larger marginal profit must have smaller $(r_i - \pi)s_i$, which further implies a smaller s_i and u_i. The next lemma further demonstrates the relationship between two competitive sellers.

Lemma A.4 **(Relation between two competitive sellers)**. *Suppose seller i and seller j are both in competitive set $\mathcal{C}$ such that $i < j$. Then, there exists a threshold $M_{i,j} > 0$ such that*
(i) $\pi(\mathbf{x}^i(M)) \leq \pi(\mathbf{x}^j(M))$ *for* $M \leq M_{i,j}$; *and*
(ii) $\pi(\mathbf{x}^i(M)) > \pi(\mathbf{x}^j(M))$ *for* $M > M_{i,j}$.

Proof of Lemma A.4. Suppose seller i and seller j are both in competitive set $\mathcal{C}$ such that $i < j$. Due to the labelling of $\mathcal{C}$, $r_i > r_j > \pi$. By definition of $G(\cdot)$ (see equation (A.1)), $\pi(\mathbf{x}^i(M)) > \pi(\mathbf{x}^j(M))$ (resp. $\pi(\mathbf{x}^i(M)) \leq \pi(\mathbf{x}^j(M))$) is equivalent to $G(\mathbf{x}^i(M)) > G(\mathbf{x}^j(M))$ (resp. $G(\mathbf{x}^i(M)) \leq G(\mathbf{x}^j(M))$), which is true for $M > {((r_j - \pi)s_j - (r_i - \pi)s_i)}/{(r_i - r_j)s_is_j}$ (resp. $M \leq {((r_j - \pi)s_j - (r_i - \pi)s_i)}/{(r_i - r_j)s_is_j}$). Define $M_{i,j} \doteq {((r_j - \pi)s_j - (r_i - \pi)s_i)}/{(r_i - r_j)s_is_j}$. Note that $M_{i,j} > 0$ since $r_i > r_j$ and $(r_j - \pi)s_j > (r_i - \pi)s_i$ by Lemma A.3. □

This lemma shows that since seller i and seller j do not dominate each other: which one of $\pi(\mathbf{x}^i(M))$ and $\pi(\mathbf{x}^j(M))$ is larger depends on the available budget M. Recall that $i < j$ represents $r_i > r_j$ and $u_i < u_j$. When budget M is limited, i.e., $M < M_{i,j}$, the seller with larger basic utility is more profitable, i.e., $\pi(\mathbf{x}^i(M)) < \pi(\mathbf{x}^j(M))$. As budget M increases, eventually, the seller with higher marginal profit is more profitable, i.e., $\pi(\mathbf{x}^i(M)) > \pi(\mathbf{x}^j(M))$ for $M > M_{i,j}$. Following the same logic, we characterize the optimal marketing strategy $\mathbf{x}^*(M)$ in two extreme cases in the next lemma.

Lemma A.5 **(Optimal marketing strategy in extreme cases)**. *When M is large enough, $\mathbf{x}^1(M)$ is optimal, i.e., $\mathbf{x}^*(M) = \mathbf{x}^1(M)$. When M is small enough, $\mathbf{x}^m(M)$ is optimal, i.e., $\mathbf{x}^*(M) = \mathbf{x}^m(M)$.*

Proof of Lemma A.5. By Lemma A.4, $\mathbf{x}^1(M)$ is optimal if $M > \max_{j\in[N]} M_{1,j}$, and $\mathbf{x}^m(M)$ is optimal if $M < \min_{j\in[N]} M_{j,m}$, where $M_{1,j} > 0$ for all $j \in [N]$ and $M_{j,m} > 0$ for all $j \in [N]$. □

When M is large enough, the optimal marketing strategy $\mathbf{x}^*(M)$ concentrates on the competitive seller with the largest marginal profit, i.e., $\mathbf{x}^*(M) = \mathbf{x}^1(M)$. When M is close to 0, the optimal marketing strategy $\mathbf{x}^*(M)$ concentrates on competitive seller with largest basic utility u_i, i.e., $\mathbf{x}^*(M) = \mathbf{x}^m(M)$.

Can there exist a seller $i \in C$ that it is never optimal to concentrate on regardless of the value of M? The answer is "yes", as demonstrated by the following lemma.

Lemma A.6 **(Consequence of joint dominance)**. *Suppose seller $i, j, k \in \mathcal{C}$ satisfy $i < j < k$ and*

$$r_j s_j < r_i s_i \frac{s_k - s_j}{s_k - s_i} + r_k s_k \left(1 - \frac{s_k - s_j}{s_k - s_i}\right). \tag{A.2}$$

Then, $\mathbf{x}^j(M)$ cannot be optimal for any $M > 0$.

Proof of Lemma A.6. Suppose seller $i, j, k \in \mathcal{C}$ satisfy $i < j < k$ and equation (A.2). By Lemma A.4, $\pi(\mathbf{x}^j(M)) > \pi(\mathbf{x}^i(M))$ if and only if $M < M_{i,j}$ and $\pi(\mathbf{x}^j(M)) > \pi(\mathbf{x}^k(M))$ if and only if $M > M_{j,k}$. Hence, $\pi(\mathbf{x}^j(M)) > \max\{\pi(\mathbf{x}^i(M)), \pi(\mathbf{x}^k(M))\}$ if and only if $M \in (M_{j,k}, M_{i,j})$. However, equation (A.2) is equivalent to $M_{i,j} < M_{j,k}$, i.e., $(M_{j,k}, M_{i,j})$ is empty. Therefore, $\mathbf{x}^j(M)$ is not optimal for any $M > 0$ because for any $M > 0$, $\pi(\mathbf{x}^j(M)) < \max\{\pi(\mathbf{x}^i(M)), \pi(\mathbf{x}^k(M))\}$. □

Note that $i < j < k$ implies $s_i < s_j < s_k$, which further implies ${(s_k - s_j)}/{(s_k - s_i)} \in (0, 1)$. Thus, equation (A.2) states that the revenue $r_j s_j$ of seller j is smaller than the weighted average revenue (i.e., right-hand side of equation (A.2)) of seller i and k, where the weight is given by ${(s_k - s_j)}/{(s_k - s_i)}$. Lemma A.6 shows that if a seller $j \in \mathcal{C}$ is jointly dominated by sellers $i, k \in \mathcal{C}$ according to Definition 2, then $\mathbf{x}^j(M)$ is not optimal for any $M > 0$. The intuition is that seller j can never be the best among the three sellers i, j, k. We provide an example (see Example EC.1) to show that joint dominance is possible.

Example EC.1 (Joint dominance). Suppose there are three sellers with marginal profits vector $(r_1, r_2, r_3) = (6.3, 5.99, 5.7)$ and basic utility vector $(u_1, u_2, u_3) = (0, 0.56, 2.4)$. Then, all sellers are competitive sellers, but seller 2 is jointly dominated by seller 1 and seller 3. Hence, it is never optimal to concentrate on seller 2.

We are ready to prove the statement in Theorem 1.

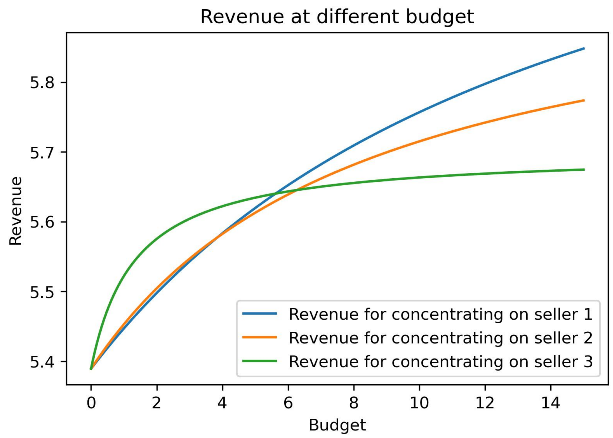

Figure EC.1 **Seller 2 is jointly dominated by seller 1 and 3, but not dominated by neither seller 1 nor seller 3. Thus, it can never be optimal even seller 2 is a competitive seller.**

Proof of Theorem 1. Firstly, we show that seller i is optimal for some $M > 0$ if and only if seller $i \in \mathcal{D}$. By Lemmas A.1, A.2 and A.6, for any seller $i \notin \mathcal{D}$, $\mathbf{x}^i(M)$ is not optimal for any $M > 0$. Let seller $j \in \mathcal{C}$ be a competitive seller that is also in the candidate set $\mathcal{D}$. By Lemma A.4, $\mathbf{x}^j(M)$ is optimal if and only if $M < M_{i,j}$ for all $i \in \mathcal{C}$ such that $i < j$ and $M > M_{j,k}$ for all $k \in \mathcal{C}$ such that $k > j$, i.e., $M \in (\max_{k>j,k\in\mathcal{C}} M_{j,k}, \min_{i<j,i\in\mathcal{C}} M_{i,j})$. We claim this interval is not empty. To see this, since seller $j \in \mathcal{D}$ is not jointly dominated by any pair of sellers $i, j \in \mathcal{C}$, as shown in the proof of Lemma A.6, $(M_{j,k}, M_{i,j})$ is not empty. Thus, the interval $(\max_{k>j,k\in\mathcal{C}} M_{j,k}, \min_{i<j,i\in\mathcal{C}} M_{i,j}) = \cap_{i<j<k;i,k\in\mathcal{C}}(M_{j,k}, M_{i,j})$ is not empty neither. Let $\mu_{i,l}$ denote $\max_{k>j,k\in\mathcal{C}} M_{j,k}$ and $\mu_{i,r}$ denote $\min_{i<j,i\in\mathcal{C}} M_{i,j}$. Hence, for each seller $i \in \mathcal{D}$, there exists some nonempty interval $I_i \doteq (\mu_{i,l}, \mu_{i,r})$ such that $\mathbf{x}^i(M)$ is optimal if and only if $M \in I_i$.

Notice that for any $i, j \in \mathcal{D}$, $I_i \cap I_j = \emptyset$ since $\pi(\mathbf{x}^i(M)) > \pi(\mathbf{x}^j(M))$ for any $M \in I_i$. This further implies either I_i is on the left of I_j (i.e., $\mu_{i,l} < \mu_{i,r} < \mu_{j,l} < \mu_{j,r}$) or I_j is on the left of I_i (i.e., $\mu_{j,l} < \mu_{j,r} < \mu_{i,l} < \mu_{i,r}$). Next, we show that for seller $i, j \in \mathcal{D}$ such that $i < j$, the interval I_j is on the left of interval I_i, i.e., $\mu_{j,l} < \mu_{j,r} < \mu_{i,l} < \mu_{i,r}$. For the sake of contradiction, suppose $\mu_{i,l} < \mu_{i,r} < \mu_{j,l} < \mu_{j,r}$. Let $M' > 0$ be a number in I_i. Then, $\mathbf{x}^i(M')$ is optimal. By Lemma A.4, we have $M' > M_{i,j}$, which further implies $\pi(\mathbf{x}^i(M)) > \pi(\mathbf{x}^j(M))$ for any $M \geq M' > M_{i,j}$. This contradicts the fact that every $M \in (\mu_{j,l}, \mu_{j,r})$ satisfies $M > M'$ and $\pi(\mathbf{x}^j(M)) > \pi(\mathbf{x}^i(M))$.

Finally, we show that $\mu_{i,l} = \mu_{i+1,r}$. For the sake of contradiction, suppose $\mu_{i+1,r} < \mu_{i,l}$. Let M' be a number such that $\mu_{i+1,r} < M' < \mu_{i,l}$. Suppose $\mathbf{x}^k(M')$ is optimal for some $k \in \mathcal{D}$, i.e., $M' \in I_k$. Then, we have (i) $k \neq i$ and $k \neq i+1$ since $k \notin I_i$ and $k \notin I_{i+1}$; and (ii) I_k is at the middle of I_i and I_{i+1}, i.e., $\mu_{i+1,r} \leq \mu_{k,l} < \mu_{k,r} \leq \mu_{i,l}$. If $k < i$, $\mu_{k,r} < \mu_{i+1,l}$, which is a contradiction since we have already shown that I_{i+1} should be on the left of I_k for $k < i+1$. If $k > i+1$, then $\mu_{k,l} > \mu_{i,r}$, which is a contradiction, since we have already shown that I_k should be on the left of I_i for $k > i$. Therefore, $\mu_{i,l} = \mu_{i+1,r}$ and we can define $\mu_i \doteq \mu_{i,l} = \mu_{i+1,r}$ for all $i \in \mathcal{D}$.

In conclusion, there exists a sequence of threshold $\{\mu_k\}_{0\le k\le|\mathcal{D}|}$ with $\infty=\mu_0>\mu_1>\ldots>\mu_{|\mathcal{D}|-1}>\mu_{|\mathcal{D}|}=0$ such that seller i is optimal if and only if $M\in(\mu_i,\mu_{i-1})$. $\square$

EC.1.3. Proof of Corollary 1

In the case that $r_i\equiv r$ for all $i\in[N]$, $R_i>R_j$ if and only if $u_i>u_j$. Therefore, by Proposition 1, optimal marketing strategy concentrates all effort on the seller with the highest u_i.

EC.1.4. Proof of Proposition 2

To begin with, we show the following structure for the optimal marketing strategy.

Lemma A.7 **(Concentrated optimal solution)**. *For any optimal solution $(\mathbf{p}^*,\mathbf{x}^*)$ to Problem 3, the marketing strategy $\mathbf{x}^*$ is concentrated.*

Proof. By Proposition 1, for any given prices $\mathbf{p}$, the optimal marketing strategy is generically unique and concentrated. Therefore, given the price vector $\mathbf{p}^*$, the optimal marketing strategy $\mathbf{x}^*$ is generically unique and concentrated. $\square$

Next, we cite the following classical result about optimal prices of standard MNL pricing problem in the literature (Li and Huh 2011). Recall that a_i represents the offering's intrinsic quality, b_i represents the price sensitivity parameter, and c_i represents the marginal cost. The standard MNL pricing problem is defined as

$$\max_{\mathbf{p}}\sum_{i\in[N]}(p_i-c_i)\frac{e^{a_i-b_ip_i}}{1+\sum_{j\in[N]}e^{a_j-b_jp_j}}.$$

Lemma A.8 **(Optimal prices of standard MNL pricing problem (Li and Huh 2011))**. *In the standard MNL pricing problem, the optimal expected profit ρ^* is the unique value of ρ satisfying*

$$\rho=\sum_{i\in[N]}\frac{e^{a_i-b_ic_i-1}}{b_i}e^{-b_i\rho}.$$

Furthermore, the optimal prices vector $\mathbf{p}^*$ satisfies

$$p_i^*-c_i-\frac{1}{b_i}=\rho^*,\quad\forall i\in[N].$$

Let $\pi(\mathbf{p},\mathbf{x})$ be the total profit given pricing strategy $\mathbf{p}$ and marketing strategy $\mathbf{x}$. If we fix marketing strategy $\mathbf{x}(M)$ and choose prices $\mathbf{p}$ to maximize the total profit $\pi(\mathbf{p},\mathbf{x}(M))$, then this is a standard MNL pricing problem. By Lemma A.8, the optimal prices vector is unique. Define this unique solution as $\mathbf{p}(\mathbf{x}(M))\doteq\arg\max_{\mathbf{p}}\pi(\mathbf{p},\mathbf{x}(M))$. Moreover, as shown in literature (Li and Huh 2011), the optimal prices of a standard MNL pricing problem can be easily computed by a bisection search algorithm. Thus, $\mathbf{p}(\mathbf{x}(M))$ can be easily computed by a bisection search algorithm for any given $\mathbf{x}(M)$. On the other hand, if we fix prices $\mathbf{p}$ and choose marketing strategy $\mathbf{x}(M)$ to maximize the total profit $\pi(\mathbf{p},\mathbf{x}(M))$, then the optimal marketing strategy is one of $\{\mathbf{x}^i(M)\}_{i\in[N]}$ characterized by Theorem 1. Therefore, to solve the non-convex Problem 3, we only need to check at most N pairs of solutions $(\mathbf{p}(\mathbf{x}^i(M)), \mathbf{x}^i(M))$, and each pair can be computed efficiently. We summarize this result formally in Proposition 2 and prove it as follows.

Proof of Proposition 2. By Lemma A.7, the marketing strategy $\mathbf{x}^*$ of any jointly optimal solution $(\mathbf{p}^*, \mathbf{x}^*)$ to Problem 3 is concentrated. Moreover, as explained above, fixing any marketing strategy $\mathbf{x}$, the corresponding profit-maximization price vector $\mathbf{p}(\mathbf{x})$ is unique. Combining these facts together, we can conclude the joint optimal solutions to Problem 3 must be obtained among the pairs $(\mathbf{p}(\mathbf{x}^i(M)), \mathbf{x}^i(M))$, i.e., $\pi(\mathbf{p}^*(M), \mathbf{x}^*(M)) = \max_{i\in[N]} \pi(\mathbf{p}(\mathbf{x}^i(M)), \mathbf{x}^i(M))$.

Recall that $\mathbf{x}^i(M)$ concentrates all budget on seller i, i.e., $\mathbf{x}^i(M)_i = \ln(M+1)$ and $\mathbf{x}^i(M)_j = 0$ for all $j \neq i$. Given marketing strategy $\mathbf{x}^i(M)$, the corresponding profit-maximization price vector $p(\mathbf{x}^i(M))$ is the optimal solution to the following standard MNL pricing problem:

$$\max_{\mathbf{p}} \frac{\sum_{k\neq i}(p_k - c_k)e^{a_k - b_k p_k} + (p_i - c_i)e^{a_i + \ln(M+1) - b_i p_i}}{1 + \sum_{j\neq i} e^{a_j - b_j p_j} + e^{a_i + \ln(M+1) - b_i p_i}}.$$

Therefore, by Lemma A.8, the price vector $\mathbf{p}(\mathbf{x}^i(M))$ and profit value $\pi(\mathbf{p}(\mathbf{x}^i(M)), \mathbf{x}^i(M))$ satisfy the properties stated in Proposition 2. □

EC.1.5. Proof of Theorem 2

Firstly, we identify the qualification of sellers to be optimal, i.e., requirements on a_i, b_i, and c_i to guarantee that there exists some $M > 0$ such that seller i is optimal.

Recall in Proposition 2, the value $\pi(\mathbf{p}(\mathbf{x}^i(M)), \mathbf{x}^i(M))$ is obtained by solving the following equation of ρ:

$$\rho = \sum_{k\in[N]} \frac{e^{a_k - b_k c_k - 1}}{b_k} e^{-b_k \rho} + M \frac{e^{a_i - b_i c_i - 1}}{b_i} e^{-b_i \rho}. \tag{A.3}$$

Define the right hand side of equation (A.3) as function $h_i(\rho, M) : [0, \infty) \times [0, \infty) \to (0, \infty)$.

Lemma A.9. *For seller $i, j \in [N]$ such that $b_i < b_j$, define $\rho_{i,j} \doteq \frac{(a_j - b_j c_j - \ln b_j) - (a_i - b_i c_i - \ln b_i)}{b_j - b_i}$. Then, $\rho_{i,j}$ is the unique solution to the equation $h_i(\rho, M) = h_j(\rho, M)$ for any $M > 0$.*

Proof. $\rho_{i,j}$ is the only solution to the equation $h_i(\rho, M) = h_j(\rho, M)$ for any $M > 0$. □

Recall that π_0 represents the optimal total profit when $M = 0$, i.e., there is no marketing strategy and pricing is the only decision. For convenience, in this proof, we also define $\pi_i(M) \doteq \pi(\mathbf{p}(\mathbf{x}^i(M)), \mathbf{x}^i(M))$. The next lemma shows that if a seller j is dominated by some other seller according to Definition 3, then for any $M > 0$, seller j is not optimal.

Lemma A.10 **(Consequence of dominance)**. *Suppose Assumption 1 holds. If seller i dominates seller j, then $\pi_i(M) > \pi_j(M)$ for any $M > 0$. Specifically, if seller j is dominated by some seller i, $(\mathbf{p}(\mathbf{x}^j(M)), \mathbf{x}^j(M))$ cannot be optimal for any $M > 0$.*

Proof. Suppose seller i dominates seller j. We consider two cases: (i) $b_i < b_j$; and (ii) $b_i = b_j$.

Case (i): Suppose $b_i < b_j$. By Lemma A.9, recall that $\rho_{i,j}$ is the intersection point of $h_i(\rho, M)$ and $h_j(\rho, M)$, i.e., $h_i(\rho_{i,j}, M) = h_j(\rho_{i,j}, M)$. Then, $\rho_{i,j} \leq \pi$ by the Definition 3. We make the following observations: (i) $\rho_{i,j}$ does not depend on M; (ii) $h_i(\rho, M) > h_j(\rho, M)$ for all $\rho > \rho_{i,j}, M > 0$; (iii) $h_i(\rho, 0) = h_j(\rho, 0)$ for all $\rho > 0$.

By Lemma A.8, π is the unique solution to the equation $\rho = h_i(\rho, 0)$. Similarly, by Proposition 2, $\pi_i(M)$ (resp. $\pi_j(M)$) is the unique solution to the equation $\rho = h_i(\rho, M)$ (resp. $\rho = h_j(\rho, M)$). Since $\rho_{i,j} \leq \pi$ and $h_k(\rho, 0)$ is decreasing in ρ for any $k \in [N]$, we have $h_i(\rho_{i,j}, 0) = h_j(\rho_{i,j}, 0) \geq h_i(\pi, 0) = \pi$. This further implies that for all $M > 0$, $h_i(\rho_{i,j}, M) = h_j(\rho_{i,j}, M) > h_i(\rho_{i,j}, 0) = h_j(\rho_{i,j}, 0) \geq \pi$, since $h_i(\rho, M)$ is increasing in M for any $\rho > 0, i \in [N]$. Hence, $h_i(\rho_{i,j}, M) = h_j(\rho_{i,j}, M) > \rho_{i,j}$ for any $M > 0$, which further implies $\pi_i(M) > \rho_{i,j}$ and $\pi_j(M) > \rho_{i,j}$. Since $h_i(\rho, M) > h_j(\rho, M)$ for all $\rho > \rho_{i,j}, M > 0$, this implies $\pi_i(M) = h_i(\pi_i(M), M) > h_j(\pi_j(M), M) = \pi_j(M)$ for all $M > 0$.

Case (ii): Suppose $b_i = b_j$. Then, by Definition 3, we have $a_i - b_i c_i > a_j - b_j c_j$. (Notice that by Assumption 1, $a_i - b_i c_i \neq a_j - b_j c_j$ for sure.) This implies $h_i(\rho, M) > h_j(\rho, M)$ for any $\rho > 0$ given $M > 0$. Again, by Proposition 2, $\pi_i(M)$ (resp. $\pi_j(M)$) is the unique solution to the equation $\rho = h_i(\rho, M)$ (resp. $\rho = h_j(\rho, M)$). This implies $\pi_i(M) > \pi_j(M)$ for all $M > 0$. $\square$

Particularly, when sellers have identical price sensitivity parameters, i.e., $b_i \equiv b$, for all $i \in [N]$, a direct implication of Definition 3 is shown as follows.

Corollary A.1. *Suppose sellers have identical price sensitivity parameters, i.e., $b_i \equiv b$ for all $i \in [N]$. Let i^* be the seller with the largest $a_i - b_i c_i$. For any $M > 0$, $(\mathbf{p}(\mathbf{x}^{i^*}(M)), \mathbf{x}^{i^*}(M))$ is the unique optimal solution to Problem 3.*

Proof. In this case, seller i^* dominates all other sellers by Definition 3. Thus, there is a unique optimal solution, which is $(\mathbf{p}(\mathbf{x}^{i^*}(M)), \mathbf{x}^{i^*}(M))$ by Proposition 2 and Lemma A.10. $\square$

By Lemma A.10, given any $M > 0$, to identify the joint optimal solution $(\mathbf{p}^*(M), \mathbf{x}^*(M))$, we only need to check sellers that are not dominated by any other sellers. The competitive set $\mathcal{C}_2$ defined in Definition 3 is never empty. This is because optimal sellers for some $M > 0$ are always in it since $\mathcal{C}_2$ is defined by a necessary condition for $\mathbf{x}^*(M)$. Next lemma demonstrates the relation among competitive sellers.

Lemma A.11 **(Relation among competitive sellers)**. *Competitive sellers satisfy $a_1 - b_1 c_1 - \ln b_1 < \ldots < a_m - b_m c_m - \ln b_m$.*

Proof. For the sake of contradiction, suppose there exists a pair of sellers $i, j \in \mathcal{C}_2$ such that $i < j$ and $a_i - b_i c_i - \ln b_i \geq a_j - b_j c_j - \ln b_j$. Due to the labeling method of the competitive set $\mathcal{C}_2$, $b_i < b_j$. This implies that $(a_j - b_j c_j - \ln b_j) - (a_i - b_i c_i - \ln b_i) \leq 0 < (b_j - b_i)\pi$. Hence, seller i dominates seller j. This contradicts the fact that sellers i, j are competitive sellers. $\square$

The next lemma further demonstrates the relationship between two competitive sellers.

Lemma A.12 **(Relation between two competitive sellers)**. *Suppose seller i and seller j are both in competitive set $\mathcal{C}$ such that $i < j$. Then, there exists a threshold $M_{i,j} > 0$ such that* *(i)* $\pi_i(M) \le \pi_j(M)$ *for* $M \le M_{i,j}$*; and*

(ii) $\pi_i(M) > \pi_j(M)$ *for* $M > M_{i,j}$.

Proof. Suppose seller i and seller j are both in competitive set $\mathcal{C}_2$ such that $i<j$. This implies $b_i < b_j$. By Lemma A.9, $\rho_{i,j}$ is the intersection point of $h_i(\rho, M)$ and $h_j(\rho, M)$, i.e., $h_i(\rho_{i,j}, M) = h_j(\rho_{i,j}, M)$. Notice that $\rho_{i,j}$ does not depend on M, i.e., $h_i(\rho_{i,j}, M) = h_j(\rho_{i,j}, M)$ for any $M \ge 0$. Let $M_{i,j}$ be the unique solution to the following equation of M: $h_i(\rho_{i,j}, M) = \rho_{i,j}$. Solving the equation, we get $M_{i,j} = (\rho_{i,j} - \sum_{k\in[N]} e^{u_k - b_k\rho_{i,j}})\frac{b_i}{e^{a_i - b_i c_i - 1}\rho_{i,j}}$.

For $M > M_{i,j}$, $h_i(\rho_{i,j}, M) = h_j(\rho_{i,j}, M) > \rho_{i,j}$. By Proposition 2, $\pi_i(M) = h_i(\pi_i(M), M)$ and $\pi_j(M) = h_i(\pi_j(M), M)$ for any $M > 0$. Since $h_i(\rho, M) > \rho$ (resp. $h_j(\rho, M) > \rho$) for $\rho < \pi_i(M)$ (resp. $\rho < \pi_j(M)$), we have $\pi_i(M) > \rho_{i,j}$ and $\pi_j(M) > \rho_{i,j}$. Since $h_i(\rho, M) > h_j(\rho, M)$ for any $\rho > \rho_{i,j}$, $\pi_i(M) > \pi_j(M)$.

For $M < M_{i,j}$, $h_i(\rho_{i,j}, M) = h_j(\rho_{i,j}, M) < \rho_{i,j}$. In this case, we have $\pi_i(M) < \rho_{i,j}$ and $\pi_j(M) < \rho_{i,j}$. Since $h_i(\rho, M) < h_j(\rho, M)$ for any $\rho < \rho_{i,j}$, $\pi_i(M) < \pi_j(M)$. □

Lemma A.12 shows that with large enough marketing budget M, the seller with smaller price sensitivity parameter b_i is always better, regardless of intrinsic quality a_i and production cost c_i. Lemma A.12 also implies that the jointly optimal solution to Problem 3 is unique.

LEMMA A.13 **(Uniqueness)**. *The jointly optimal solution to Problem 3 is generically unique if* $M \ne M_{i,j}$ *for all* $i, j \in \mathcal{C}_2$.

Proof. Since $h_i(\rho, M)$ is strictly increasing in M, $M_{i,j}$ is the unique solution to the equation of M: $h_i(\rho_{i,j}, M_{i,j}) = \rho_{i,j}$. If $M \ne M_{i,j}$ for all $i, j \in \mathcal{C}_2$, the unique solution ρ_i to the following equation of ρ: $\rho = h_i(\rho, M)$ does not equal to $\rho_{i,j}$ for any pairs of sellers $i, j \in \mathcal{C}_2$. This implies $\rho_i \ne \rho_j$ for all $i, j \in \mathcal{C}_2$, since $\rho_{i,j}$ is the only intersection point of $h_i(\rho, M)$ and $h_j(\rho, M)$. By Proposition 2, the optimal value to Problem 3 equals to $\max_{i\in\mathcal{C}_2} \rho_i$. Let $i^* \doteq \arg\max_{i\in\mathcal{C}_2} \rho_i$. Then, by Proposition 2, $(\mathbf{p}^{i^*}(\mathbf{x}^{i^*}(M), \mathbf{x}^{i^*}(M))$ is the generically unique optimal solution. □

Next lemma characterizes the joint optimal solution to Problem 3 in two extreme cases.

LEMMA A.14 **(Optimal pricing and marketing strategy in extreme cases)**. *When* M *is large enough,* $(\mathbf{p}(\mathbf{x}^1(M), \mathbf{x}^1(M))$ *is optimal. When* M *is small enough,* $(\mathbf{p}(\mathbf{x}^m(M), \mathbf{x}^m(M))$ *is optimal.*

Proof. By Lemma A.12, if $M > \max_{j\in\mathcal{C}_2} M_{1,j}$, $\pi_1(M) > \pi_j(M)$ for all $j \in \mathcal{C}_2$ and $j \ne 1$, i.e., $(\mathbf{p}(\mathbf{x}^1(M), \mathbf{x}^1(M))$ is optimal. If $M < \min_{j\in\mathcal{C}_2} M_{j,m}$, $\pi_m(M) > \pi_j(M)$ for all $j \in \mathcal{C}_2$ and $j \ne m$, i.e., $(\mathbf{p}(\mathbf{x}^1(M), \mathbf{x}^1(M))$ is optimal. □

Therefore, with a large enough marketing budget M, the optimal solution is to concentrate all effort on the seller with the smallest price sensitivity parameter and make pricing decisions accordingly. When the marketing budget is limited, the optimal solution is to concentrate all effort on the seller with the largest $e^{a_i - b_i c_i}/b_i$ and make pricing decisions accordingly.

We realize that not all competitive sellers can be optimal. If a competitive seller j is jointly dominated by a pair of competitive sellers i and k according to Definition 4, then seller j cannot be optimal for any $M > 0$.

LEMMA A.15 **(Consequence of joint dominance relation)**. *Suppose seller $j \in \mathcal{C}_2$ is jointly dominated by sellers $i, k \in \mathcal{C}_2$ according to Definition 4. Then, $\pi_j(M)$ cannot be optimal for any $M > 0$.*

Proof. Suppose seller $j \in \mathcal{C}_2$ is jointly dominated by sellers $i, k \in \mathcal{C}_2$. By Definition 4, $M_{i,j} < M_{j,k}$. By Lemma A.12, this implies for any $M > 0$, $\pi_j(M) < \max\{\pi_i(M), \pi_k(M)\}$. Hence, $(\mathbf{p}(\mathbf{x}^j(M), \mathbf{x}^j(M))$ cannot be optimal for any $M > 0$. □

Finally, we have narrowed down the candidates for optimal solutions to the candidate set. We formally prove each statement in Theorem 2 as follows.

Proof of Theorem 2. **Part (i):** This is proved by Lemma A.13 and Proposition 2.

Part (ii): By Lemmas A.10 and A.15, any seller $i \notin \mathcal{D}_2$ is not optimal for any $M > 0$. Suppose seller j is a competitive seller that is also in candidate set $\mathcal{D}_2$. By Lemma A.12, $(\mathbf{p}(\mathbf{x}^j(M), \mathbf{x}^j(M))$ is optimal if and only if $M \in (\max_{k>j, k\in\mathcal{C}_2} M_{j,k}, \min_{i<j, i\in\mathcal{C}_2} M_{i,j})$, which is not empty since j is not jointly dominated by any pair i, k.

Part (iii): Since the optimal solution is unique, it must satisfy the asserted property proved in Proposition 2.

Part (iv): For seller $j \in \mathcal{D}_2$, by Lemma A.12, $(\mathbf{p}(\mathbf{x}^j(M), \mathbf{x}^j(M))$ is optimal if and only if $M \in (\max_{k>j, k\in\mathcal{D}_2} M_{j,k}, \min_{i<j, i\in\mathcal{D}_2} M_{i,j})$, which is not empty since j is not jointly dominated by any pair i, k. Let $\mu_{j,l}$ denote $\max_{k>j, k\in\mathcal{D}_2} M_{j,k}$ and $\mu_{j,r}$ denote $\min_{i<j, i\in\mathcal{D}_2} M_{i,j}$. Hence, for each seller $j \in \mathcal{D}_2$, there exists some nonempty interval $I_j \doteq (\mu_{j,l}, \mu_{j,r})$ such that $(\mathbf{p}(\mathbf{x}^j(M), \mathbf{x}^j(M))$ is optimal if and only if $M \in I_j$.

Notice that for any $i, j \in \mathcal{D}_2$, $I_i \cap I_j = \emptyset$ since $\pi(\mathbf{x}^i(M)) > \pi(\mathbf{x}^j(M))$ for any $M \in I_i$. This further implies either I_i is on the left of I_j (i.e., $\mu_{i,l} < \mu_{i,r} < \mu_{j,l} < \mu_{j,r}$) or I_j is on the left of I_i (i.e., $\mu_{j,l} < \mu_{j,r} < \mu_{i,l} < \mu_{i,r}$). Next, we show that for seller $i, j \in \mathcal{D}$ such that $i < j$, the interval I_j is on the left of interval I_i, i.e., $\mu_{j,l} < \mu_{j,r} < \mu_{i,l} < \mu_{i,r}$. For the sake of contradiction, suppose $\mu_{i,l} < \mu_{i,r} < \mu_{j,l} < \mu_{j,r}$. Let $M' > 0$ be a number in I_i. Then, $(\mathbf{p}(\mathbf{x}^i(M), \mathbf{x}^i(M'))$ is optimal. By Lemma A.12, we have $M' > M_{i,j}$, which further implies $\pi_i(M) > \pi_j(M)$ for any $M \geq M' > M_{i,j}$. This contradicts the fact that every $M \in (\mu_{j,l}, \mu_{j,r})$ satisfies $M > M'$ and $\pi_j(M) > \pi_i(M)$.

Finally, we show that $\mu_{i,l} = \mu_{i+1,r}$. For the sake of contradiction, suppose $\mu_{i+1,r} < \mu_{i,l}$. Let M' be a number such that $\mu_{i+1,r} < M' < \mu_{i,l}$. Suppose $(\mathbf{p}(\mathbf{x}^k(M), \mathbf{x}^k(M'))$ is optimal for some $k \in \mathcal{D}_2$, i.e.,

$M' \in I_k$. Then, we have (i)$k \neq i$ and $k \neq i+1$ since $k \notin I_i$ and $k \notin I_{i+1}$; and (ii) I_k is at the middle of I_i and I_{i+1}, i.e., $\mu_{i+1,r} \leq \mu_{k,l} < \mu_{k,r} \leq \mu_{i,l}$. If $k < i$, $\mu_{k,r} < \mu_{i+1,l}$, which is a contradiction since we have already shown that I_{i+1} should be on the left of I_k for $k < i+1$. If $k > i+1$, then $\mu_{k,l} > \mu_{i,r}$, which is a contradiction, since we have already shown that I_k should be on the left of I_i for $k > i$. Therefore, $\mu_{i,l} = \mu_{i+1,r}$ and we can define $\mu_i \doteq \mu_{i,l} = \mu_{i+1,r}$ for all $i \in \mathcal{D}_2$.

In conclusion, there exists a sequence of threshold $\{\mu_k\}_{0 \leq k \leq |\mathcal{D}_2|}$ with $\infty = \mu_0 > \mu_1 > \ldots > \mu_{|\mathcal{D}_2|-1} > \mu_{|\mathcal{D}_2|} = 0$ such that seller i is optimal if and only if $M \in (\mu_i, \mu_{i-1})$.

Part (v): As shown in Corollary A.1, in this case, the unique optimal marketing strategy concentrates on seller i with the largest $a_i - bc_i$. By Proposition 2, the (unique) optimal solution must be the pair $(\mathbf{p}^i(M), \mathbf{x}^i(M))$, and the optimal price vector $\mathbf{p}^i(M)$ satisfies the equal-markup property. □

EC.2. Proofs of Section 4

EC.2.1. Proof of Theorem 3

Proof of Theorem 3. Due to Assumption 2, any feasible solution to Problem 4 must allocate some marketing effort to some sellers in the protected group, otherwise the fairness constraint would be violated.

Since Problem 5 is a linear program, it admits a generically unique optimal basic feasible solution $\mathbf{q}^*$. That is, there should be at least N constraints that are tight at $\mathbf{q}^*$.

Firstly, we show that the budget constraint is tight at $\mathbf{q}^*$. For the sake of contradiction, suppose the budget constraint is not tight, i.e., $\sum_{i \in [N]} e^{-u_i} q_i^* < (1 - \sum_{i \in [N]} q_i^*)(M+N)$. This is equivalent to $\sum_{i \in [N]} (e^{-u_i} + M + N) q_i^* < M + N$. Suppose we increase the value of q_i^* for some $i \in P$ by a small amount $\epsilon > 0$. Denote the new solution as $\mathbf{q}'$. The fair market share constraint is still feasible since $(1-\beta) \sum_{i \in P} q_i' > (1-\beta) \sum_{i \in P} q_i^* \geq \beta \sum_{i \notin P} q_i^* = \beta \sum_{i \notin P} q_i'$. With the same reasoning as in the proof of Proposition 1, all the other constraints are feasible at $\mathbf{q}'$ and the objective value is strictly larger. This contradicts the fact that $\mathbf{q}^*$ is optimal. Thus, the budget constraint must be tight at $\mathbf{q}^*$, i.e., $\sum_{i \in [N]} e^{-u_i} q_i^* = (1 - \sum_{i \in [N]} q_i^*)(M+N)$.

With the same reasoning in the proof of Proposition 1, we conclude that at optimal solution $\mathbf{q}^*$

(i) $\sum_{i \in [N]} q^* < 1$;
(ii) $q_i^* > 0$ for all $i \in [N]$;
(iii) not all non-negative marketing effort constraints are tight.

Therefore, either 1 or 2 non-negative marketing effort constraints are not tight.

Case (i): Only 1 non-negative marketing effort constraint is not tight. This could happen if the fair market share constraint is not tight at $\mathbf{q}^*$, then Problem 5 is equivalent to Problem 2. Thus, by Proposition 1, the generically unique optimal marketing strategy $\mathbf{x}^*$ concentrates on one seller.

Moreover, by Assumption 2, this seller must be in P, otherwise the fair market share constraint is violated.

If there are 2 non-negative marketing effort constraints are not tight, one of them must corresponds to a seller in P, otherwise the fair market share constraint is violated by Assumption 2. Thus, one of the following two cases could happen.

Case (ii): Two non-negative marketing effort constraints are not tight. These two constraints correspond to a seller $i \in P$ and a seller $j \notin P$, respectively.

Case (iii): Two non-negative marketing effort constraints are not tight. These two constraints correspond to two sellers $i, j \in P$.

If a non-negative marketing constraint which corresponds to seller i is tight at $\mathbf{q}^*$, then by the inverse marketing effort function (3), $x_i^* = 0$. On the other hand, if a non-negative marketing constraint which corresponds to seller i is not tight at $\mathbf{q}^*$, then by the inverse marketing effort function (3), $x_i^* > 0$.

Next, we show that case (iii) is impossible if the budget M is large enough. Recall $\mathbf{x}^i(M)$ is the marketing strategy which concentrates on seller i, i.e., $x^i(M)_i = \ln(M+1)$ and $x^i(M)_j = 0$ for all $j \neq i$. Suppose M is large enough such that $\sum_{i\in P} s_i(\mathbf{x}^k(M)) \geq \beta \sum_{i\in[N]} s_i(\mathbf{x}^k(M))$ for all $k \in P$. That is, $\mathbf{x}^k(M)$ is feasible for any $k \in P$. Consider the variant that we only allocate effort among protected sellers. The optimal solution is to concentrate all effort on one protected seller. Thus, it is not optimal to split marketing budget between two protected sellers. $\square$

EC.2.2. Proofs of Theorem 4

Firstly, we show the following implication of Assumption 3.

LEMMA B.1. *Suppose Assumption 3 holds. Then, the fair market share constraint* (6) *is strictly satisfied by the marketing strategy which concentrates on seller* $i_P^\circ \doteq \arg\max_{i\in P} u_i$, *i.e.,*

$$\frac{Me^{u_{i^\circ}} + \sum_{i\in P} e^{u_i}}{1 + Me^{u_{i^\circ}} + \sum_{j\in[N]} e^{u_j}} > \beta \frac{Me^{u_{i^\circ}} + \sum_{i\in[N]} e^{u_i}}{1 + Me^{u_{i^\circ}} + \sum_{j\in[N]} e^{u_j}}.$$

Proof. Consider a variant of Problem 1 that we maximize the total market share of sellers in the protected group P (without fair market share constraint (6)). With the same argument in Corollary 1, the optimal marketing strategy $\mathbf{x}^*$ to this problem is to concentrate on seller i_P°. Notice that by the definition of market share in MNL model, the total market share of non-protected sellers (i.e., sellers not in P) is minimized at $\mathbf{x}^*$. By Assumption 3, there exists a feasible marketing strategy $\mathbf{x}'$ to Problem 4 that strictly satisfies the fair market share constraint (6). Then, by the optimality of $\mathbf{x}^*$, we have $(1-\beta)\sum_{i\in P} q_i(\mathbf{x}^*) \geq (1-\beta)\sum_{i\in P} q_i(\mathbf{x}') > \beta \sum_{i\notin P} q_i(\mathbf{x}') \geq \beta \sum_{i\notin P} q_i(\mathbf{x}^*)$. Equivalently, fair market share constraint (6) is strictly satisfied at $\mathbf{x}^*$. $\square$

Proof of Theorem 4. Case (i): Consider the relaxed maximization problem without the fair market share constraint. Then this is the same problem studied in Corollary 1. By Corollary 1, the optimal solution is to concentrate all effort on i_P° since seller i_P° has the largest basic utility. This solution is feasible even with the fair market share constraint. Thus, it is still optimal.

Case (ii): Firstly, we show that it is not optimal to only allocate effort among sellers in P. Consider the following variant:

$$\begin{aligned} \max_{\mathbf{x}\geq 0} \quad & \frac{\sum_{i\in P} e^{u_i+x_i} + \sum_{j\notin P} e^{u_j}}{1+\sum_{i\in P} e^{u_i+x_i} + \sum_{j\notin P} e^{u_j}} \\ \text{s.t.} \quad & \sum_{i\in P}(e^{x_i}-1) \leq M. \end{aligned}$$

Following the same argument in the proof of Proposition 1, the optimal solution $\mathbf{x}^*$ to this problem is to concentrate all marketing effort on seller i_P° since seller i_P° has the largest basic utility in P. By Assumption 3, we know that the fair market share constraint is satisfied and not tight at $\mathbf{x}^*$. Therefore, if we slightly transfer some effort from i_P° to i_R°, the solution is feasible to Problem 4. Specifically, consider the following new marketing strategy $\mathbf{x}'$: $y'_{i_P^\circ} = e^{x'_{i_P^\circ}} - 1 = M - \epsilon$, $y'_{i_R^\circ} = e^{x'_{i_P^\circ}} - 1 = \epsilon$, and $y'_j = e^{x'_j} - 1 = 0$ for all $j \neq i_P^\circ, i_R^\circ$. If ϵ is small enough, $\mathbf{x}'$ is feasible to Problem 4. However, the total market share $\sum_{i\in[N]} s_i(\mathbf{x}')$ of $\mathbf{x}'$ is larger than the total market share $\sum_{i\in[N]} s_i(\mathbf{x}^*)$ of $\mathbf{x}^*$, i.e., $\sum_{i\in[N]} s_i(\mathbf{x}') > \sum_{i\in[N]} s_i(\mathbf{x}^*)$. To see this,

$$\sum_{i\in[N]} s_i(\mathbf{x}') = \frac{(M-\epsilon)e^{u_{i_P^\circ}} + \epsilon e^{u_{i_R^\circ}} + \sum_{i\in[N]} e^{u_i}}{(M-\epsilon)e^{u_{i_P^\circ}} + \epsilon e^{u_{i_R^\circ}} + 1 + \sum_{i\in[N]} e^{u_i}} > \frac{Me^{u_{i_P^\circ}} + \sum_{i\in[N]} e^{u_i}}{Me^{u_{i_P^\circ}} + 1 + \sum_{i\in[N]} e^{u_i}} = \sum_{i\in[N]} s_i(\mathbf{x}^*), \quad \text{(B.1)}$$

since we assume $u_{i_P^\circ} < u_{i_R^\circ}$ in this case. Hence, in Problem 4, it is not optimal to only allocate marketing effort among sellers in P. On the other hand, by Assumption 2, it is not feasible to only allocate marketing effort among sellers not in P. Thus, at the optimal marketing strategy to Problem 4, there is at least one seller in P and one seller not in P who would get a positive marketing effort.

Similar to the variant (B.1), consider the problem that we only allocate an arbitrary amount of marketing effort not in P. With the same argument, the optimal solution is to concentrate all available effort on i_R°. Therefore, the optimal solution to Problem 4 is to split all marketing effort between i_P° and i_R°.

Note that in this case, the fair market share constraint is tight at the optimal solution. To see this, for the sake of contradiction, suppose the fair market share constraint is not tight at the optimal solution. Then, the problem is equivalent to the problem studied in Corollary 1. Thus, the optimal solution is to concentrate all marketing effort on seller i_R° since this seller has the largest basic utility. However, this solution violates the fair market share constraint due to Assumption 2, which is a contradiction.

Since both the budget constraint and fair market share constraints are tight, we get the exact value of $e^{x_{i_P^\circ}} - 1$ by solving the equation system of tight budget constraint and fair market share constraint. □

EC.2.3. Proof of Corollary 2

Proof of Corollary 2. Case (i): from Theorem 4, given optimal marketing strategy $\mathbf{x}^*$ to Problem 4,

$$e^{x_{i_P^\circ}} - 1 = \frac{\beta(\sum_{j\in[N]} e^{u_j} + Me^{u_{i_R^\circ}}) - \sum_{j\in P} e^{u_j}}{(1-\beta)e^{u_{i_P^\circ}} + \beta e^{u_{i_R^\circ}}},$$

which is smaller than M and decreasing in u_{i_P} for $u_{i_P^\circ} < u_{i_R^\circ}$.

Case (ii): in this case, the optimal marketing strategy concentrates all effort on i_P by Theorem 4. □

EC.3. Proofs of Section 5

EC.3.1. Proofs of Section 5.2.1

To begin with, we show the following KKT condition for Problem 7.

LEMMA C.1. *For $\alpha \in (0,1)$, given optimal solution $\mathbf{q}^*$ to Problem 7, there exist constants $\mu_i \geq 0$ for all $i = 0,\dots,N$ such that*

$$q_i^* = [\frac{r_i^{1-\alpha}}{e^{-u_i}(\mu_0 - \mu_i) + (M+N)\mu_0 - \sum_{j\in[N]} \mu_j}]^{\frac{1}{\alpha}}, \quad \forall i \in [N].$$

For $\alpha = 1$, there exist constants $\mu_i \geq 0$ for all $i = 0,\dots,N$ such that

$$q_i^* = \frac{1}{e^{-u_i}(\mu_0 - \mu_i) + (M+N)\mu_0 - \sum_{j\in[N]} \mu_j}, \quad \forall i \in [N].$$

Moreover, $\mu_i = 0$ if seller i gets positive marketing effort.

Proof. Since Problem 7 is a convex optimization problem, we can apply the Karush-Kuhn-Tucker (KKT) conditions as follows. For $\alpha \in (0,1)$, at optimal solution $\mathbf{q}^*$, there exist constants $\mu_i \geq 0$ for $i = 0,\dots,N$, such that

$$-(r_i q_i^*)^{-\alpha} r_i + \mu_0(e^{-u_i} + M + N) - \mu_i(e^{-u_i} + 1) - \sum_{j\neq i} \mu_j = 0, \quad \forall i \in [N],$$

$$\mu_0(\sum_{j\in[N]} e^{-u_j} q_j^* - (1 - \sum_{j\in[N]} q_j^*)(M+N)) = 0,$$

$$\mu_i(1 - \sum_{j\in[N]} q_j^* - e^{-u_j} q_j^*) = 0, \quad \forall i \in [N].$$

By complementary slackness condition, i.e., the third equation, $\mu_i = 0$ if the non-negative marketing effort constraint corresponding to seller i is not tight at $\mathbf{q}^*$, i.e., $\mathbf{x}_i^* > 0$ by the inverse marketing effort function (3). Similarly, for $\alpha = 1$, the KKT condition gives us the stated results. □

Proof of Proposition 3. To begin with, we reformulate the problem by transforming $y_i = e^{x_i} - 1$ as follows

$$\max_{\mathbf{y}\geq \mathbf{0}} \quad \frac{\sum_{i\in[N]}[r_i(y_i+1)e^{u_i}]^{1-\alpha}}{[1+\sum_{j\in[N]}(y_j+1)e^{u_j}]^{1-\alpha}}$$
$$\text{s.t.} \quad \sum_{i\in[N]} y_i \leq M.$$

Specifically, when $\alpha = 1$, the objective is equivalent to $\sum_{i\in[N]}\ln(y_i+1) - N\ln(1+\sum_{j\in[N]}(y_j+1)e^{u_j})$.

Part (i): Let $\mathbf{y}^*$ be the optimal solution. For the sake of contradiction, suppose $y_i^* < y_j^*$ for some $i<j$. Let $\Delta = y_j^* - y_i^*$ be the difference. Then, since $u_i \leq u_j$, notice that $\ln(y_i^*+1)+\ln(y_j^*+1) = \ln(y_i^*+\Delta+1)+\ln(y_j^*-\Delta+1)$ and

$$N\ln(1+\sum_{k\in[N]}(y_k^*+1)e^{u_k}) \geq N\ln(1+(y_i^*+\Delta+1)e^{u_i}+(y_j^*-\Delta+1)e^{u_j}+\sum_{k\neq i,j}(y_k^*+1)e^{u_k}).$$

Therefore, the objective value becomes strictly larger if we set y_i^* to be $y_i^*+\Delta$ and y_j^* to be $y_j^*-\Delta$, which contradicts the optimality of $\mathbf{y}^*$.

Part (ii): This is implied by part (i).

Part (iii): Let $\mathbf{y}^*$ be the optimal solution. It suffices to show that $y_k^* > 0$ if M is large enough, for any $k\in[N]$. If this is true, then by part (ii), as M increases, the largest index of a seller that is allocated a positive amount of marketing effort increases from 1 to N.

For the sake of contradiction, suppose $y_k^* = 0$. By part (i), we know that $y_1^* \geq y_2^* \geq \ldots \geq y_k^* = 0$ and $y_j^* = 0$ for all $j\geq k$. Since $\sum_{i\in[N]} y_i^* = M$, this implies $y_1^* \geq M/k$. Suppose we transfer Δ marketing effort from seller 1 to seller k. Define

$$f(\Delta) \doteq \ln(y_1^*-\Delta+1)+\ln(\Delta+1) - N\ln(1+(y_1^*-\Delta+1)e^{u_1}+(\Delta+1)e^{u_k}+\sum_{j\neq 1,k}(y_j^*+1)e^{u_j}).$$

The derivative at $\Delta = 0$ is

$$f'(0) = 1 - \frac{1}{y_1^*+1} - N\frac{e^{u_k}-e^{u_1}}{1+(y_1^*+1)e^{u_1}+e^{u_k}+\sum_{j\neq 1,k}(y_j^*+1)e^{u_j}},$$

which increases to 1 as y_1^* goes to infinity. Since $y_1^* \geq M/k$, if M is large enough, $f'(0) > 0$. This implies the objective is larger if we transfer a little effort Δ from seller 1 to seller k, which contradicts the optimality of $\mathbf{y}^*$.

Moreover, since $y_1^* \geq M/k$ and $\sum_{j\in[N]}(y_j^*+1)e^{u_j} \geq e^{u_1}(\sum_{j\in[N]} y_j^* + N) = (M+N)e^{u_1}$, we have

$$\frac{1}{y_1^*+1} + N\frac{e^{u_k}-e^{u_1}}{1+(y_1^*+1)e^{u_1}+e^{u_k}+\sum_{j\neq 1,k}(y_j^*+1)e^{u_j}} \leq \frac{1}{\frac{M}{k}+1} + N\frac{e^{u_k}-e^{u_1}}{e^{u_1}(M+N)},$$

which is strictly smaller than 1 if M is large enough. That is, when the budget is large enough, it cannot be optimal to only allocate effort among sellers 1 to $k-1$.

Part (iv): If M is sufficiently large, by Proposition 3, all sellers are allocated positive marketing effort at optimal marketing strategy, i.e., $\mathbf{x}_i^* > 0$ for all $i\in[N]$. By Lemma C.1, this implies $\mu_i = 0$ for $i = 1,\ldots,N$. In this case, $q_i^* = 1/\mu_0(e^{-u_i}+M+N)$ for all $i\in[N]$. This implies the stated ratios. □

EC.3.2. Proofs of Section 5.2.2

Proof of Theorem 5. To begin with, we reformulate the problem by transforming $y_i = e^{x_i} - 1$ as follows

$$\begin{aligned} \max_{\mathbf{y}\geq \mathbf{0}} \quad & \frac{\sum_{i\in[N]}[r_i(y_i+1)e^{u_i}]^{1-\alpha}}{[1+\sum_{j\in[N]}(y_j+1)e^{u_j}]^{1-\alpha}} \\ \text{s.t.} \quad & \sum_{i\in[N]} y_i \leq M. \end{aligned} \tag{C.1}$$

Part (i): Let $\mathbf{y}^*$ be the optimal solution. For the sake of contradiction, suppose $y_i^* < y_j^*$ for some $i<j$. Set both y_i^* and y_j^* as $\bar{y} = (y_i^* + y_j^*)/2$. Firstly, we observe that the denominator of the objective function decreases. Next, the change of the numerator $r_i^{1-\alpha}e^{(1-\alpha)u_i}[(\bar{y}+1)^{1-\alpha} - (y_i^*+1)^{1-\alpha}] - r_j^{1-\alpha}e^{(1-\alpha)u_j}[(y_j^*+1)^{1-\alpha} - (\bar{y}+1)^{1-\alpha}]$ is positive if and only if

$$\frac{r_i}{r_j}e^{u_i-u_j} > \left(\frac{(y_j^*+1)^{1-\alpha} - (\bar{y}+1)^{1-\alpha}}{(\bar{y}+1)^{1-\alpha} - (y_i^*+1)^{1-\alpha}}\right)^{\frac{1}{1-\alpha}}.$$

Since the right-hand side converges to 0 as α converges to 1, the inequality holds when α is large enough. Thus, the objective function increases when α is large enough. This contradicts the optimality of $\mathbf{y}^*$.

Part (ii): This is implied by part (i).

Part (iii): Let $\mathbf{y}^*$ be the optimal solution. It suffices to show that $y_k^* > 0$ if M is large enough, for any $k \in [N]$. If this is true, then by part (ii), as M increases, the largest index of a seller that is allocated a positive amount of marketing effort increases from 1 to N.

For the sake of contradiction, suppose $y_k^* = 0$. By part (i), we know that $y_1^* \geq y_2^* \geq \ldots \geq y_k^* = 0$ and $y_j^* = 0$ for all $j \geq k$. Since $\sum_{i\in[N]} y_i^* = M$, this implies $y_1^* \geq M/k$. Suppose we transfer Δ marketing effort from seller 1 to seller k. The α-fairness as a function of transfer Δ is given by

$$f(\Delta) \doteq \frac{[r_1(y_1^*-\Delta+1)e^{u_1}]^{1-\alpha} + [r_k(\Delta+1)e^{u_k}]^{1-\alpha} + \sum_{j\neq 1,k}[r_j(y_j^*+1)e^{u_j}]^{1-\alpha}}{[1+(y_1^*-\Delta+1)e^{u_1} + (\Delta+1)e^{u_k} + \sum_{j\neq 1,k}(y_j^*+1)e^{u_j}]^{1-\alpha}}.$$

Then, $f'(0) > 0$ is equivalent to

$$\frac{(r_k e^{(1-\alpha)u_k} - r_1(y_1^*+1)^{-\alpha}e^{(1-\alpha)u_1})[1+(y_1^*+1)e^{u_1} + e^{u_k} + \sum_{j\neq 1,k}(y_j^*+1)e^{u_j}]}{[r_1(y_1^*+1)e^{u_1}]^{1-\alpha} + r_k^{1-\alpha}e^{(1-\alpha)u_k} + \sum_{j\neq 1,k}[r_j(y_j^*+1)e^{u_j}]^{1-\alpha}} > e^{u_k} - e^{u_1},$$

which holds if y_1^* is large enough since left hand side converges to $+\infty$ as y_1^* goes to infinity. Since $y_1^* \geq M/k$, if M is large enough, $f'(0) > 0$. This implies the objective is larger if we transfer a little effort Δ from seller 1 to seller k, which contradicts the optimality of $\mathbf{y}^*$.

Moreover, a lower bound for the left-hand side is

$$\frac{((r_{\min}e^{u_k})^{1-\alpha} - (\frac{M}{k}+1)^{-\alpha}(r_{\max}e^{u_k})^{1-\alpha})(\frac{M}{k}+1)e^{u_1}}{N(\frac{M}{k}+1)^{1-\alpha}(r_{\max}e^{u_k})^{1-\alpha}}.$$

This is larger than $e^{u_k} - e^{u_1}$ if $M > (\frac{r_{\max}}{r_{\min}})^{\frac{1-\alpha}{\alpha}} N[N(e^{u_k-u_1}-1)+1]^{\frac{1}{\alpha}}$.

Part (iv): If M is sufficiently large, by part (iii), all sellers are allocated positive marketing effort at optimal marketing strategy, i.e., $\mathbf{x}_i^* > 0$ for all $i \in [N]$. By Lemma C.1, this implies $\mu_i = 0$ for $i = 1, \ldots, N$. In this case, $q_i^* = (r_i^{1-\alpha}/\mu_0(e^{-u_i}+M+N))^{\alpha}$ for all $i \in [N]$. This implies the stated ratios. $\square$

EC.4. Experimental setup in Section 6

The marginal profit vector of 5 sellers is $(r_1, r_2, r_3, r_4, r_5) = (2.5, 3, 3.5, 2, 1)$, and the basic utility vector is $(u_1, u_2, u_3, u_4, u_5) = (5, 4.5, 4, 3.5, 3)$. The total budget is $M = 55$.